
# AppOpt 2016

XII International Conference
Approximation and Optimization
in the Caribbean

# BOOK OF ABSTRACTS

Havana University, CUBA June 5–10, 2016

# Contents

## Aims and scope of the conference


This conference is the twelve of a series, dedicated to research on Approximation and Optimization. The first two meetings were held in Havana (Cuba) in 1987 and 1993. Since then, these meetings have been organized in the following countries of the Caribbean area: Puebla (México) 1995, Caracas (Venezuela) 1997, Pointe-à-Pitre (Guadeloupe) 1999, Guatemala City (Guatemala) 2001, León (Nicaragua) 2004, Santo Domingo (República Dominicana) 2006, San Andrés (Colombia) 2008, San Salvador (El Salvador) 2011, and Puebla (México) 2013.

The goal of these conferences is to support the development of high level research and education in the Caribbean. Included are: invited lectures, tutorials, mini-symposia, and contributed talks on the following topics:

**Approximation:** Wavelets, polynomial and rational approximation, splines, orthogonal polynomials, interpolation, asymptotic analysis, radial basis functions, numerical methods.

**Optimization:** Continuous and discrete optimization, parametric, stochastic and global optimization, nonlinear equations and inequalities, nonsmooth analysis, critical point theory, control theory.

**Mathematical economics:** Fixed point theory, equilibria of competitive economies, portfolio problems, cooperative and non-cooperative games.

**Applications:** Engineering and energy models, robotics, pattern recognition, image restoration, applications in biology, economy and science.

# General Program

| **VENUE** | Havana University, Building "Felipe Poey"<br>Faculty of Mathematics and Computing Science | |
|---|---|---|
| **Time** | **Activity** | **Place** |
| **Monday, 6** | | |
| 08:00 - 12:00 | REGISTRATION | Salón de Reuniones, Building "Felipe Poey" |
| 09:00 - 10:00 | INVITED LECTURE: V. Totik | Classroom 6 |
| 10:00 - 10:20 | SNACK | |
| 10:30 - 11:30 | WORK SESSIONS | Classrooms |
| 12:00 - 13:30 | LUNCH | "La Roca" Restaurant |
| 14:00 - 14:45 | OPENING CEREMONY | Aula Magna |
| 15:00 - 17:30 | WORK SESSIONS | Classrooms |
| 18:00 - 19:30 | WELCOME COCKTAIL | |
| **Tuesday, 7** | | |
| 09:00 - 10:00 | INVITED LECTURE: C. Clason | Classroom 6 |
| 10:00 - 10:20 | SNACK | |
| 10:30 - 12:00 | WORK SESSIONS | Classrooms |
| 12:30 - 14:00 | LUNCH | "La Roca" Restaurant |
| 14:30 - 17:30 | WORK SESSIONS | Classrooms |
| **Wednesday, 8** | FREE | |
| **Thursday, 9** | | |
| 09:00 - 10:00 | INVITED LECTURE: J. S. Geronimo | Classroom 6 |
| 10:00 - 10:20 | SNACK | |
| 10:30 - 12:00 | WORK SESSIONS | Classrooms |
| 12:30 - 14:00 | LUNCH | "La Roca" Restaurant |
| 14:30 - 17:30 | WORK SESSIONS | Classrooms |
| 20:00 - 23:00 | FAREWELL DINNER | |
| **Friday, 10** | | |
| 09:00 - 10:00 | INVITED LECTURE: V. Shikhman | Classroom 6 |
| 10:00 - 10:20 | SNACK | |
| 10:30 - 12:00 | WORK SESSIONS | Classrooms |
| 12:30 - 14:00 | LUNCH | "La Roca" Restaurant |
| 14:30 - 17:30 | WORK SESSIONS | Classrooms |

# Committees and Invited Speakers

### Executive Committee

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### Invited Plenary Speakers

Christian Clason
Duisburg-Essen University, Germany.

Jeffrey S. Geronimo
Georgia Institute of Technology, USA.

Vladimir Shikhman
KU Leuven, Belgium.

Vilmos Totik
University of South Florida, USA & University of Szeged, Hungary.

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Havana, Cuba
1987, 1993 & 2016
Puebla, México
1995 & 2013
Pointe-à-Pitre
Guadeloupe
1999
Santo Domingo
Republica Dominicana
2006
Guatemala City, Guatemala
2001
San Salvador, El Salvador
2011
León, Nicaragua
2004
San Andrés
Colombia
2008
Caracas, Venezuela
1997
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# Plenary Talks

## Nonsmooth optimization of partial differential equations

Christian Clason
Duisburg-Essen University, Germany

**Abstract**

This talk is concerned with the analysis and numerical solution of optimization problems involving non-differentiable functionals and equality constraints given by partial differential equations. Such problems arise in, e.g. optimal switching controls of distributed systems or parameter identification problems with data corrupted by impulsive noise. Using tools of nonsmooth analysis, explicit pointwise optimality conditions can be derived which are amenable to numerical solution via generalized Newton or hybrid gradient methods.

## Bivariate Bernstein-Szegő weights on the square

Jeffrey S. Geronimo
Georgia Institute of Technology, Atlanta, USA.

**Abstract**

Univariate Bernstein-Szegő weights, which are weights whose densities are the reciprocal of a positive polynomial, play an important role in the theory of orthogonal polynomials by providing a convenient set of simple approximating weights to a general measure. Recently there has been work on extending these types of measures to the bicircle. I will review this work and describe extensions in the case of the square.

## Algorithmic principle of least revenue for finding market equilibria

Vladimir Shikhman
Department of Computer Science, KU Leuven, Belgium.

**Abstract**

In analogy to extremal principles in physics, we introduce the Principle of Least Revenue for treating market equilibria. It postulates that equilibrium prices minimize the total excessive revenue of market's participants. As a consequence, the necessary optimality conditions describe the clearance of markets, i.e. at equilibrium prices supply meets demand. It is crucial for our approach that the potential function of total excessive revenue be convex. This facilitates structural and algorithmic analysis of market equilibria by using convex optimization techniques. In particular, results on existence, uniqueness, and efficiency of market equilibria follow easily. The market decentralization fits into our approach by the introduction of trades or auctions. For that, Duality Theory of convex optimization applies. The computability of market equilibria is ensured by applying quasi-monotone subgradient methods for minimizing nonsmooth convex objective - total excessive revenue of the market's participants. We give an explicit implementable algorithm for finding market equilibria which corresponds to real-life activities of market's participants.

## The asymptotic form of the Gauss-Lucas theorem

Vilmos Totik
University of South Florida, Tampa, USA.
University of Szeged, Hungary.

**Abstract**

The Gauss-Lucas theorem claims that if the zeros of a polynomial P lie in a convex set $K$, then the zeros of its derivative also lie in $K$. This is no longer true if a zero of P may lie outside $K$, but it was conjectured by Boris Shapiro that then most of the zeros of $P'$ still lie in any fixed (arbitrarily small) neighborhood of $K$. (The precise conjecture was that if most of the zeros of P lie in K, then most of the zeros of its derivative lie in any fixed neighborhood of $K$.) In this talk we give a proof and discuss the connection to some old results of Erdös, Niven, de Bruijn and Springer and to their later developments.

# Short Talks

# Discrete integrable systems generated by Angelesco systems of Hermite-Padé aproximants

Alexander I. Aptekarev, Keldysh Institute of Applied Mathematics

**Abstract**

In the series of papers [1], [2], [3] a general approach to multidimensional difference operators with discrete integrable potentials related to the Hermite-Padé approximants was developed. In our talk the electromagnetic Schrödinger operator in $l_2(\mathbb{Z}_+^d)$ will be considered. The potential satisfies to a discrete integrable system related with multiple orthogonal polynomials with respect to the Angelesco system of measures. The problem on limits of the potential along the rays in $\mathbb{Z}_+^d$ will be discussed. A statement and solution of the scattering problem is considered as well.

These results are obtained jointly with S. A. Denisov and M. L.Yattselev and announced in [4].

**References**

1. A. I. Aptekarev, M. Derevyagin, W. van Assche, Discrete integrable systems generated by multiple orthogonal polynomials, **284** (2014), 168-191. (arXiv: 1409.4053)
2. A. I. Aptekarev, M. Derevyagin, W. van Assche, On 2D discrete Schrödinger operators associated with multiple orthogonal polynomials, J. Phys. A: Math. Theor., **48** (2015), 065201 (16pp). (arXiv: 1410.1332)
3. A. I. Aptekarev, M. Derevyagin, H. Miki, W. van Assche, Multidimensional Toda lattices: continuous and discrete time, (arXiv: 1511.08098)
4. A. I. Aptekarev, S. A. Denisov, M. L.Yattselev, Completely integrable on $\mathbb{Z}_+^d$ potentials for electromagnetic Schrödinger operator: rays asymptotics and scattering problem, Keldysh Institute Preprints, **88** (2015), (20pp). (http://mi.mathnet.ru/eng/ipmp/y2015/p88)

# Projected solutions of Nash games with applications to electricity markets

Didier Aussel, University of Perpignan
Asrifa Sultana, University of Perpignan, IIT Madras
V. Vetrivel, IIT Madras

**Abstract**

A generalized Nash problem is a non-cooperative game in which the constraint set is depending on the variable. But the constraint map may not be a self map and then there is usually no solution. Thus we define the concept of projected solution and, based on a fixed

point theorem, we establish some results on existence of projected solution for generalized Nash games in a finite dimensional space where the constraint map is not necessarily self-map. Projected solution of quasi-variational inequalities and quasi-optimization problems will be also considered.

All these developments will be motivated by the bid process in electricity markets.

The talk is based on reference [1].

**References**

1. D. Aussel, A. Sultana and V. Vetrivel, On the Existence of Projected Solutions of Quasi-Variational Inequalities and Generalized Nash Equilibrium Problems, submitted (2015), 14 pp.

## Solvability and new primal-dual partition of the space of linear semi-infinite continuos optimization problems

Abraham B. Barragán, Benemérita Universidad Autónoma de Puebla
Lidia A. Hernández, Benemérita Universidad Autónoma de Puebla
Maxim I. Todorov, Universidad de las Américas Puebla

**Abstract**

In this paper we consider the set of all semi-infinite linear programming problems with a fixed compact set of indices and continuous right and left hand side coefficients. In the whole parameter space, the problems are classified in a different manner. F.i., in our case, consistent and inconsistent, solvable (with bounded optimal value and optimal set non-empty), unsolvable (with bounded optimal value and empty optimal set) and unbounded (i.e., with infinity optimal value), etc. Various different partitions have been considered in previous papers. In the majority of them, the classification is made for both, the primal and the dual problems. Our classification generates a partition of the parameter space, which we call second general primal-dual partition. We have tried to characterize each cell of the partition by means of necessary and sufficient and in some cases only necessary or sufficient conditions which guarantee that the pair of problems (primal and dual) belongs to the cell. In addition, we show that each cell of the partition is non empty and with examples that the conditions are only necessary or sufficient. Finally, we present a study of the stability of the presented partition.

**References**

1. Duer, M., Jargalsaikhan, B. and Still, G. *Genericity results in linear conic programming -a tour d'horizon.* Mathematics of Operations Research, to appear.
http://wwwhome.math.utwente.nl/~stillgj/sip/rev_bds.pdf.

2. Goberna M.A., Jornet V. and Puente R. *Optimización Lineal. Teoría, Métodos y Modelos* [Spanish]. Mc Graw Hill, Madrid, (2004).

3. Goberna M.A. and López M. *Linear Semi-infinite Optimization.* Wiley, Chichester, (1998).

4. Goberna M.A. and Todorov M.I. *Generic Primal-dual solvability in continuous linear semi-infinite optimization.* Optimization, Vol. 57, (2008) 239-248.

5. Goberna M.A. and Todorov M.I. *Primal-dual stability in continuous linear optimization.* Mathematical Programming, Vol. 116B, (2009) 129-147.

6. Ochoa P.D. and Vera de Serio V.N. *Stability of the primal-dual partition in linear semi-in.nite programming.* Optimization, Vol. 61, (2012) 1449-1465.

# Best rational approximation to functions with finitely many singularities

Laurent Baratchart, INRIA
Maxim Yattselev, IUPUI Indianapolis

**Abstract**

Let $f$ be analytic of a complex variable in the complement of the unit disk and continuous on its closure. Assume that $f$ can be meromorphically continued inside the disk, except over finitely many points that may be algebraic branchpoints or essential singularities (branchpoints of infinite order are not allowed). We call the class of such functions $\mathcal{A}$.

To each $f \in \mathcal{A}$, we associate a compact set $\mathcal{K}_f$, included in the open unit disk $\mathbb{D}$, outside of which $f$ is single-valued and which has minimum Green capacity in $\mathbb{D}$ among all such compact sets. The existence of $\mathcal{K}_f$ and its uniqueness up to a set of capacity zero were proven in [1] by the late H. Stahl, who co-authored the present research. We put $\mathrm{cap}(\mathbb{D}, \mathcal{K}_f)$ for the Green capacity of $\mathcal{K}_f$.

For $n \geq 1$ an integer, we let $r_n(f)$ be a best approximant to $f$ in the *sup*-norm on the complement of $\mathbb{D}$, out of rational functions of degree at most $n$. We also let

$$e_n(f, r_n(f)) := \sup_{|z|\geq 1} |f(z) - r_n(f)(z)|$$

be the optimal error. When $f \in \mathcal{A}$ has no essential singularities, it follows from the work by A. Gonchar and E. Rakhmanov [2] that

$$\lim_{n\to\infty} e_n^{1/n}(f, r_n(f)) = \exp\left\{-\frac{2}{\mathrm{cap}(\mathbb{D}, \mathcal{K}_f)}\right\}. \tag{1}$$

In this talk, we will report on the following result:

When $f \in \mathcal{A}$, then (1) holds. Moreover, if $f$ has branchpoints, then $\mathrm{cap}(\mathbb{D}, \mathcal{K}_f)) > 0$ and the normalized counting measure of the poles of $r_n(f)$ converges weak-*, as $n \to \infty$, to

the Green equilibrium distribution on $\mathcal{K}_f$. In fact, any sequence $\rho_n$ of rational functions of degree at most $n$ such that

$$\lim_{n\to\infty} e_n^{1/n}(f,\rho_n)) = \exp\left\{-\frac{2}{\mathrm{cap}(\mathbb{D},\mathcal{K}_f)}\right\}$$

has normalized counting measure of poles that converges weak-* to the Green equilibrium distribution on $\mathcal{K}_f$.

The proof dwells on AAK theory and potential theory.

**References**

1. H. Stahl, Extremal domains associated with an analytic function, I, II, Complex Variables Theory Appl. **4** (1985), 311-324 & 325-338.
2. A. A. Gonchar and E. A. Rakhmanov, Equilibrium distributions and the degree of rational approximation of analytic functions, Mat. Sb. **134(176)** (1987), 306-352.

# The Darboux transformations and the Kostant-Toda system

D. Barrios Rolanía, Universidad Politécnica de Madrid

**Abstract**

In [1] some aspects of the relation between the $(p+2)$-banded matrices

$$J = \begin{pmatrix} a_{0,0}(t) & 1 & & & & \\ a_{1,0}(t) & a_{1,1}(t) & 1 & & & \\ \vdots & \vdots & \ddots & \ddots & & \\ a_{p,0}(t) & a_{p,1}(t) & \cdots & a_{p,p}(t) & 1 & \\ 0 & a_{p+1,1}(t) & & \ddots & \ddots & \ddots \\ & 0 & \ddots & & & \\ & & \ddots & & & \end{pmatrix} \tag{2}$$

and the integrable system

$$\dot{a}_{i,j}(t) = (a_{i,i}(t) - a_{j,j}(t))\, a_{i,j}(t) + a_{i+1,j}(t) - a_{i,j-1}(t)\,, \qquad i,j = 0,1,\ldots \tag{3}$$

were studied. In particular, a method for constructing solutions of this system was given in the case $p = 2$. This method is based in the extension of the concept of Darboux transformation, which can be consulted in [3] for the classical tridiagonal case $p = 1$. Due to the matrical interpretation of this method, the concept of transform of Darboux was extended in

[1] for an arbitrary $p \in \mathbb{N}$ and a banded matrix $J$ as in (2). However, just in [2] the existence of such kind of generalized transformation was determined. As a consequence, now we have the appropriate conditions to generalize the method for constructing solutions of (3), given in [1] in the case $p = 2$, to any $p \in \mathbb{N}$. This is precisely the object of this paper.

**References**

1. D. Barrios Rolanía, A. Branquinho, A. Foulquié Moreno, *On the full Kostant-Toda system and the discrete Korteweg-de Vries equations*, J. Math. Anal. Appl. **401** (2013), 811-820.
2. D. Barrios Rolanía, D. Manrique, On the existence of Darboux transformations for banded matrices, Applied Mathematics and Computation **253** (2015), 116-125.
3. M.I. Bueno, F. Marcellán, Darboux transformation and perturbation of linear functionals, Linear Algebra and its Applications **384** (2004), 215-242.

## Bergman orthogonal polynomials and the Grunsky matrix

Bernhard Beckerman, University of Lille

**Abstract**

By exploiting a link between Bergman orthogonal polynomials and the Grunsky matrix probably first observed by Kühnau in 1985, we improve some recent results on strong asymptotics of Bergman polynomials outside the domain $G$ of orthogonality, and on entes of the Bergman shift. We also recall from the literature some links between regularity of the boundary of $G$, the Grunsky operator and the related conformal maps.

Joint work with Nikos Stylianopoulos.

## On proximal (sub)gradient splitting method for nonsmooth convex optimization problems

Yunier Bello Cruz, IME, Federal University of Goiás

**Abstract**

In this talk we present a variant of the proximal forward-backward splitting iteration for solving nonsmooth optimization problems in Hilbert spaces, when the objective function is the sum of two nondifferentiable convex functions. The proposed iteration, which will be called Proximal Subgradient Splitting Method, extends the classical subgradient iteration for important classes of problems, exploiting the additive structure of the objective function. The weak convergence of the generated sequence was established using different stepsizes and under suitable assumptions. Moreover, we analyze the complexity of the iterates.

# Random processes and minimal energies in high dimensional spheres

Carlos Beltrán, Universidad de Cantabria

**Abstract**

In this talk I will present advances in different aspects of the problem of distributing points in spaces which satisfy some symmetry conditions, such as high-dimensional spheres.

- On one hand, two new determinantal point processes in the $d$–sphere will be presented. One of them is valid for any dimension and another only for even dimensions. In both cases, the number of points that are drawn in the sphere from these processes can be carried out for an infinite collection of natural numbers. Many interesting results about these processes will be mentioned, remarkably the fact that the expected value of Riesz's energy improves the known bounds for the minimum energy of points in the sphere, getting the constant in the second term in the assymptotic expansion closer to the conjecture by Brauchart, Hardin and Saff [4].

- On the other hand, a general result for the so–called locally harmonic manifolds (a concept that corresponds to some extent with a natural notion of symmetry) will be presented that shows how the heat equation defines naturally a *heat potential* and a *heat energy*. These concepts can be used in the general theory and when specified to the sphere case in any dimension and number of points, an interesting relation with separation, discrepancy and the usual Riesz potentials arrises.

Different parts of this work are common work with different coauthors: Nuria Corral, Juan G. Criado del Rey, Ujué Etayo, Jordi Marzo and Joaquim Ortega–Cerdá.

**References**

1. C. Beltrán, J. Marzo, J. Ortega–Cerdá *Energy and discrepancy of rotationally invariant determinantal point processes in high dimensional spheres.* To appear, see `http://arxiv.org/abs/1511.02535`.
2. C. Beltrán, N. Corral, J. G. Criado del Rey *Distributing heat sources in the hypersphere.* To appear.
3. C. Beltrán, U. Etayo *Work in progress.*
4. J. S. Brauchart, D. P. Hardin, E. B. Saff. *The next-order term for optimal Riesz and logarithmic energy asymptotics on the sphere.* Recent advances in orthogonal polynomials, special functions, and their applications, 31-61, Contemp. Math., 578, Amer. Math. Soc., Providence, RI, 2012.

# A new approach to the proximal point method: convergence on general Riemannian manifolds

Glaydston de Carvalho Bento, Federal University of Goiás
João X. da Cruz Neto, Federal University of Piauí
Paulo R. Oliveira, Federal University of Rio de Janeiro

**Abstract**

In my talk, I will go present an approach to the proximal point method in the Riemannian context, which was introduced, in this setting, by Ferreira and Oliveira (Optimization 51(2), 257-270, 2002). In particular, without requiring any restrictive assumptions about the sign of the sectional curvature of the manifold, I will talk about full convergence for any bounded sequence generated from the proximal point method, in the case that the objective function satisfies the Kurdyka-Lojasiewicz inequality. This approach extends the applicability of the proximal point method to be able to solve any problem that can be formulated as the minimizing of a definable function, such as one that is analytic, restricted to a compact manifold on which the sign of the sectional curvature is not necessarily constant; see [1].

**References**

1. G. C. Bento, J. X. Cruz Neto, P. R. Oliveira, A New Approach to the Proximal Point Method: Convergence on General Riemannian Manifolds, J. Optim. Theory Appl., **168** (2016), 743-755.

# Optimizing derivatives through stochastic programming

Carlos N. Bouza Herrera, Universidad de La Habana
Sira Allende Alonso, Universidad de La Habana
Daniel Chen, Smith & King College

**Abstract**

In many environment applications, the objective of the analysis is not to predict the entire derivative curve of a conditional expectation function at each data point. Instead, it is often sufficient to construct statistics to estimate the Average Derivative. Commonly the analysis is based on a regression function. We will be concerned about the point-wise consistency of an estimator of a density function.

A common solution is using nonparametric density estimation. Hence many applications need exploring the estimation of a density for working with a good approximation to the underlying model.

Under a set of suitable conditions on the objective function we need of the asymptotic convergence of a kernel density estimator.

This paper is concerned with modeling the optimization of the estimation of a derivative using a suitable stochastic programming model. The key problem is that using an estimate of the density poses the need of evaluating quantitatively the effect of solving the approximate problem. That is analyzing the behavior of the approximation error.

We develop a study of the related issues and establish the bounding of the approximation error. The pointwise converge of kernel estimator of the density ensures that the involved metrics tend to zero. Therefore having a sufficiently large sample size enable to be confident that the estimated optimal solution is close to the real solution.

**References**

1. Bouza, C., Approximate solutions of stochastic linear programs with estimated objective function: an application of re-sampling methods. Rev.Bras.Biom. 26, (2008),115-130.
2. Bouza, C. and Allende, S.: Density estimation and the approximation error in Stochastic Linear Programming Inv. Operacional, 10 (1989), 135-148.
3. Gine, E. and Guillou, A., Rates of strong uniform consistency for multivariate kernel density estimators, Annales de l'Institut Henri Poincaré (B) Probability and Statistics, 38, (2002), 907 921.
4. Houda, M., Wasserstein metrics and empirical distributions in stability of stochastic programs. In Lukáčik M. (ed.): Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XII).Bratislava, University of Economics, (2004), 7177.
5. Kaňková, V. and Houda M., A note on quantitative stability and empirical estimates in stochastic programming. In Leopold-Wildburger, U., Rendl, F., Wäsher, G. (eds.): Operations Research Proceedings 2002. Heidelberg, Springer-Verlag, 413418

# Scalar functions for computing minimizers under variable order structures

Gemayqzel Bouza Allende, Universidad de La Habana
Christianne Tammer, Martin Luther University

**Abstract**

Let $K : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ be a set-valued mapping. A point $a^*$ is a minimal element of the set $A \subset Y$ with respect to the ordering structure induced by $K$ if there is not $a \in A$, $a^* \neq a$, such that $a^* - a \in K(a^*)$. This model appears in economic and in medical practical problems, see [1], [3], [6]. Characterizations of minimal points and related classes of points as well as algorithms for computing them can be found in [2], in particular with the aid of scalarizing functions, i.e. real valued functions whose zeros are closely related with the set of minimizers. In this work we will extend the scalarizing function studied in [4], [5] to the variable order case. We study its properties and analyze its applications to the practical

computation of solutions.

**References**


1. Baatar, D., and M.M. Wiecek, Advancing equitability in multiobjective programming, Comput. Math. Appl.**2** (2006), 225-234.
2. Eichfelder, G.,Variable ordering structure in in vector optimization,Springer, (2014).
3. Engau, A.,Variable preference modeling with ideal-symmetric convex cones, J. Global Optim. **42** (2008), 295– 311.
4. Gerstewitz, Ch., Nichtkonvexe Dualitat in der Vektoroptimierung, Wiss. Z. Tech. Hochsch, Leuna-Merseburg, **25**, 3, (1983), 357-364.
5. Gerth, C., P. Weidner, Nonconvex separation theorems and some applications in vector optimization, J. Optim. Theory Appl. **67** 2, (1990), 297-320.
6. Mordukhovich, B.S, T. Q. Bao and A. Soubeyran: Fixed points and variational principles with applications to capability theory of wellbeing via variational rationality, Set-Valued Var. Anal. **23** (2015), 375-398.


# Un algoritmo memético aplicado al problema de planificación de recorridos turísticos


Luis Alberto Bouza Álvarez, Universidad de La Habana
Dafne García de Armas, Universidad de La Habana
Alina Fernández Arias, Universidad de La Habana



**Abstract**

El problema de planificación de recorridos turísticos es una de las variantes más recientes del problema del viajante [1]. Dado un conjunto de puntos de interés (POI) y un origen, se desea diseñar un recorrido que maximice la satisfacción del turista [2]. Las limitaciones de tiempo y resistencia física impiden que se incluyan todos los puntos, luego es importante realizar una adecuada selección de los POI a visitar así como el orden en que se realiza el recorrido. En este trabajo se propone una estrategia de solución para este problema basada en un Algoritmo Memético. La población inicial se construye siguiendo el paradigma goloso - aleatorio. Se consideraron cuatro funciones de cruzamiento inspiradas en el operador *Order Crossover* y se emple la metaheurística Búsqueda por Entornos Variables Descendentes para la mutación. En la elección de los elementos sobrevivientes para la próxima generación se empleó un criterio elitista-tabú con el fin de lograr una mayor diversidad en la nueva población. Los resultados preliminares muestran la efectividad del procedimiento desarrollado.

**References**

1. D. Tas and M. Gendreau and O. Jabali and G. Laporte, The Traveling Salesman Problem with Time-Dependent Service Times, CIRRELT-2014-48, (2014).
2. B. Wu and W. Sun and Y. Murata and K. Yasumoto and M. Ito, A stamina-aware sightseeing tour scheduling method, in *Proceeding of the 13th annual conference companion on Genetic and evolutionary computation*, GECCO, (2011), 81-82.

## Numerical approximations of optimal portfolios in mispriced asymmetric Lévy markets

Winston Buckley, Bentley University
Hongwei Long, Florida Atlantic University
Mario Marshall, Western Governors University

**Abstract**

We present numerical approximations of optimal portfolios in mispriced Lévy markets under asymmetric information for informed and uninformed investors having logarithmic preference. We apply our numerical scheme to Kou [1] jump-diffusion markets by deriving analytic formulas for the first two derivatives of the underlying portfolio objective function which depend only on the Lévy measure of the jump-generating process. Optimal portfolios are then simulated using the Box-Mueller algorithm, Newton's method and incomplete Beta functions. Convergence dynamics and trajectories of sample paths of optimal portfolios for both investors are presented at different levels of information asymmetry, mispricing, horizon, asymmetry in the Kou density, jump intensity, volatility, mean-reversion speed, and Sharpe ratios.

We also apply the proposed Newton's algorithm to compute optimal portfolios for investors in Variance Gamma markets via instantaneous centralized moments of returns.

**References**

1. S. G. Kou, A jump-diffusion model for option pricing, Management Science, **48** (2002), 1086-1101.

## Interpolación de la superficie craneal para el diseño automático de prótesis

Javier Cabrera, Universidad de La Habana
Oscar L. Vera, Universidad de La Habana

**Abstract**

Tras una craneotomía o accidente severo que requiera la eliminación de parte del hueso del cráneo se suele insertar una prótesis que ocupe la zona afectada. El diseño y fabricación de dicha prótesis se realiza generalmente de forma artesanal y requiere largos períodos de tiempo de intervención quirúrgica abriendo espacios a otras complicaciones. En los últimos años se han reportado casos en los que se ha logrado, con el uso de impresoras 3D, acortar los tiempos de creación de moldes e incluso de la propia prótesis [1] pero el diseño sigue siendo manual. Algunos trabajos han intentado diseñar de forma automática la pieza a partir de imágenes de tomografía del paciente pero encuentran problemas al fijar la prótesis en el lugar de destino [2]. El presente trabajo propone, a partir de los cortes en las propias imágenes de tomografía, interpolar la zona faltante utilizando lemniscatas de pocos focos y así obtener un modelo de cráneo lo más aproximado posible al original. En cada corte se busca la curva bidimensional que mejor se adapte a la superficie del hueso considerando todos los puntos pertenecientes al cráneo, una medida que evalúe la simetría en el resultado y un factor de considere el ajuste de la curva a la zona de la lesión. La composición de las curvas obtenidas en cada corte aproximan la superficie tridimensional de la parte afectada. Este resultado constituye el punto de partida para el diseño automático de la pieza a insertar. En el trabajo se mostrarán resultados obtenidos en un conjunto de imágenes propias donde se han simulado las lesiones craneales.

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## Problema del cartero chino sobre redes mixtas

Joanna Campbell Amos, Universidad de La Habana
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**Abstract**

El Problema del Cartero Chino (CPP por sus siglas en inglés) es un tipo particular de problemas de recorridos sobre arcos y/o aristas de un grafo. Este problema tiene como objetivo encontrar un recorrido cerrado de longitud mínima, que atraviese al menos una vez todos los arcos y/o aristas de un grafo [3]. Para el problema sobre grafos dirigidos (DCPP) y grafos no dirigidos (UCPP) existen algoritmos exactos polinomiales. Sobre un grafo mixto el problema es NP-difícil [7], por lo que las heurísticas resultan una atractiva propuesta de solución aproximada.

El trabajo propone ofrecer un panorama de la situación actual de las investigaciones sobre el MCPP. Se abordan diferentes métodos de solución, tanto exactos como heurísticos [1],

[2], [3], [4], [6]. Entre estos últimos, se presenta una heurística basada en la meta heurística Recocido Simulado, propuesta por las autoras del trabajo. Elemento fundamental del algoritmo es la generación de multígrafos mediante adición de arcos y orientación de aristas, siguiendo un procedimiento de búsqueda local. De ese modo se construyen grafos dirigidos sobre los que es posible aplicar un algoritmo exacto. Se comparan los resultados obtenidos por la heurística propuesta y las implementaciones de otros algoritmos [5], [6].

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# A smoothing approach for mathematical programs with complementarity constraints


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Gemayqzel Bouza Allende, Universidad de La Habana


**Abstract**


Consider the constrained optimization problem:

$$M = \left\{ \begin{array}{rcl} \min f(x) & \text{s.t. } x \in M, & \\ h(x) = 0, & g(x) \leq 0, & \\ x \in \mathbb{R}^n : \; G(x) \geq 0, & H(x) \geq 0, & \\ G^T(x)H(x) = 0, & & \end{array} \right\} \tag{4}$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $g : \mathbb{R}^n \to \mathbb{R}^m$, $h : \mathbb{R}^n \to \mathbb{R}^p$, $G : \mathbb{R}^n \to \mathbb{R}^l$, $H : \mathbb{R}^n \to \mathbb{R}^l$ are continuously differentiable functions. Due the complementary term $G(x) \geq 0,\ H(x) \geq 0,\ G^T(x)H(x) = 0,$

are called Mathematical Programs with Complementarity Constraints (MPCC). As reported in [2] the set of feasible solutions of the MPCC has a complex structure. That is why several solution approaches have appear such as the smoothing approach studied in [1], [3]. In this paper an implementation of this method is presented. The corresponding parametric problem is solved by a predictor-corrector approach. Some numerical examples illustrate its behaviour.

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## Time and band limiting for matrix orthogonal polynomials

Mirta M. Castro Smirnova, Universidad de Sevilla

**Abstract**

In this talk we will consider examples of matrix valued orthogonal polynomials satisfying differential equations (i.e. a bispectral situation) in connection with time and band limiting. For a given family of matrix orthogonal polynomials one considers the operator of time and band limiting given by an integral operator acting on matrix valued functions, which we will denote by $K$. We then exhibit a second order differential operator $\widetilde{D}$ such that $K\widetilde{D} = \widetilde{D}K$. The existence of a commuting local operator is very useful in computing numerically the eigenfunctions of the given global operator.

Our study is motivated by the work of Claude Shannon and a series of papers by D. Slepian, H. Landau and H. Pollak at Bell Labs in the 1960's.

This is a joint work in progress with Alberto Grünbaum, University of California, Berkeley.

# Orthogonal polynomials in the solution of the problem of optimal control

Erick Manuel Delgado Moya, Universidad de La Habana
Aymée Marrero Severo, Universidad de La Habana

**Abstract**

An alternative method based on the orthogonal polynomials and their operational properties as the operational matrix of integration, product and coefficient to give a solution to the problem of getting best control is presented in this paper. The approximate numeric integration is omitted by the obtainment of a less expensive computering direct method looking for a better precision.

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# Parameter-dependent interpolatory subdivision with non-uniform parametrization

Rafael Díaz Fuentes, ICIMAF
Lucia Romani, Universitá di Milano-Bicocca

**Abstract**

Curve subdivision schemes generate smooth curves as limit of a sequence of polygons, each one obtained as a refinement of the previous one. In this work we present a family of interpolatory curve subdivision schemes where the refinement rules not only inherit the parametrization associated to the initial data, but also incorporate a global tension parameter that can be used to modify the shape of the limit curve. This constructive approach gives rise to a family of non-uniform interpolatory subdivision schemes which contains as special members most of the existing interpolatory subdivision schemes.

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# Two stochastic models related with an example coming from group representation theory

Manuel Domínguez de la Iglesia, Universidad Nacional Autónoma de México

**Abstract**

We study one example of matrix-valued orthogonal polynomials coming from the representations of the group $(\mathrm{SU}(2) \times \mathrm{SU}(2), \mathrm{diag})$ (see [1]). After appropriate transformations it is posible to obtain two stochastic models associated with this example. The first model comes from the block tridiagonal structure of the Jacobi matrix and constitute an example of a continuous-time quasi-birth-and-death process. The second model comes from the (matrix-valued) second-order differential operator and constitute an example of a switching diffusion process.

Based on a joint work with Pablo Román (Universidad Nacional de Córdoba, Argentina).

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## Hurwitz and Hurwitz-type matrices of two-way infinite series

Alexander Dyachenko, TU-Berlin

**Abstract**

A function is stable or Hurwitz-stable if all its zeros lie in the left half of the complex plane. The classical approach to the Hurwitz stability (dating back to Hermite and Biehler) exploits a deep relation between stable functions and mappings of the upper half of the complex plane into itself (i.e. $\mathcal{R}$-functions). Hurwitz introduced a connection between minors of the Hurwitz matrix and the Hankel matrix built from coefficients of the corresponding $\mathcal{R}$-function (moments), which resulted in the famous Hurwitz criterion.

More recent studies [1], [6] highlighted another property related to Hurwitz-stability: the total nonnegativity of corresponding Hurwitz matrices, that is nonnegativity of all their minors. The paper [2] extends the criterion [5] to a complete description of power series (singly infinite or finite) with totally nonnegative Hurwitz matrices. During my talk, I am going to extend this result further to two-way (i.e. doubly) infinite power series. The corresponding general case of the necessary conditions (The. 4, [4]) for total nonnegativity of generalized Hurwitz matrices follows as an application.

The study is prompted by the criterion [3], because each Hurwitz matrix is built from two Toeplitz matrices. The essential connection to Hankel matrices breaks here (no correspondent Stieltjes continued fraction), and thus the doubly infinite case requires an approach distinct from the singly infinite case.

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# Método de búsqueda local y perturbación para el problema de enrutamiento de vehículos con recogida y entrega simultánea

Alina Fernández Arias, Universidad de La Habana
Sira Allende Alonso, Universidad de La Habana

**Abstract**

El problema de enrutamiento de vehículos con recogida y entrega simultánea (VRPSPD) consiste en el diseño de rutas, con el menor costo posible, que permitan satisfacer en una única visita, las demandas de recogida y entrega de mercancía de un conjunto de clientes. Toda la mercancía se carga y descarga en el depósito central, el cuál dispone de una flota de vehículos de diferentes capacidades y costos asociados [1]-[2]. En este trabajo se presenta un enfoque por vecindades en el que se incluyen mecanismos de perturbación del óptimo local y de no obtenerse mejoras en $\mu$ iteraciones sucesivas se reinicia el proceso a partir de una solución golosa - aleatoria. Se consideraron por separado las metaheurísticas Búsqueda por Entornos Variables Descendentes (VND) que tiene carácter determinista y Recocido Simulado (SA) que es de naturaleza estocástica. Dada la complejidad asociada a la búsqueda de soluciones factibles, en este trabajo se penaliza en la función objetivo la no factibilidad por exceso de carga. La estrategia fue validada empleando el conjunto de prueba descrito por Salhi y Nagy [3]. En los experimentos realizados el procedimiento propuesto muestra mejores resultados que los obtenidos con las diferentes variantes de GRASP desarrolladas por las autoras [4]-[5]. Además se mejoraron las soluciones reportadas en la literatura en un 40% de las instancias. El comportamiento de las dos metaheurísticas sugiere el diseño un algoritmo híbrido que combine ambas estrategias.

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# Unifying the local convergence analysis of Newton's method for strongly regular generalized equations

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G. N. Silva, Universidade Federal do Oeste da Bahia

**Abstract**

In this paper we consider the Newton's method for solving the generalized equation of the form

$$f(x) + F(x) \ni 0,$$

where $f : \Omega \to Y$ is a continuously differentiable function, $X$ and $Y$ are Banach spaces, $\Omega \subseteq X$ an open set and $F : X \to Y$ be a set-valued mapping with nonempty closed graph. This generalized equation covers wide range of problems in classical analysis and its applications. For instance, if $X = R^n$, $Y = R^m$ and $F = R^s_- \times \{0\}^{m-s}$ is the product of the nonpositive orthant in $R^s$ with the origin in $R^{m-s}$, then it describes a system of $s$ inequalities and $m - s$ equalities. If $F$ is the normal cone mapping $N_C$, of a convex set $C$ in $Y$ and $Y = X^*$ the dual of $X$, the above generalized equation is the variational inequality problem; additional comments about generalized equations can be found in [1]. We show under strong regularity of the generalized equation, concept introduced by S. M. Robinson in [2] the Newton's method is locally quadratically convergent to a solution. The analysis presented based on Banach Perturbation Lemma for generalized equation and the majorant condition relaxing Lipschitz continuity of the derivative $f'$, allow to obtain the optimal convergence radius, uniqueness of solution and also unify some result pertaining the Newton's method theory.

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## Krall orthogonal polynomials on the simplex

Lidia Fernández, Universidad de Granada

**Abstract**

This research aims to analyze a Krall inner product on the simplex defined by adding a Krall term over the border to the classical inner product. Using a similar construction to Koornwinder's we get a mutually orthogonal basis with respect to this inner product and we relate it with the standard basis on the simplex. Some algebraic and analytic properties will be deduced.

This is a joint work with Antonia M. Delgado, Teresa E. Pérez and Miguel A. Piñar.

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## Consequences of Abel's theorem extended to the rows of the Padé table

Abel Fernández Infante, ISPJAE
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**Abstract**

This work responds to a suggestion made to the thesis *About the behaviour at the boundary of rows of Padé's table* defended in July 1998. The aim of this paper is to review some consequences of Abel's theorem extended to the rows of Padé's table, that show that this theorem is valid not only a wide class of functions but rather also, it can be of great utility in the practice, since like its application allows us to obtain information about the function that is represented by means of power series, beyond its convergence disk.

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# Estrategia de selección de recorridos para el problema de enrutamiento de vehículos con flota heterogénea y múltiples compartimentos

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**Abstract**

El problema de Enrutamiento de Vehículos con Flota Heterogénea y Múltiples Compartimentos (HVRP-M) [1]-[2] consiste en diseñar un sistema de rutas que permita satisfacer las múltiples demandas de un conjunto de clientes. Se dispone de una flota limitada de vehículos, diferentes en cuanto a cantidad, capacidad y tipo de producto que se pueden transportar en los compartimentos y máximo nḿero de viajes diarios. Dada la complejidad asociada a la obtención de rutas factibles, en este trabajo se propone una estrategia para la generación de múltiples recorridos factibles sobre los cuales se aplica un modelo de *Set Partition* con el que se obtienen las rutas de menor costo que satisfagan las restricciones. Para la construcción del conjunto de rutas se sigue el patrón *Greedy Randomized* con dos criterios para el cálculo del costo. En el primer caso se incluye, como próximo punto en el recorrido el cliente que minimice el costo total de la ruta. El segundo mecanismo agrega el cliente más cercano. En el conjunto de rutas creado por esta estrategia se eliminan los recorridos similares en cuanto a clientes, productos abastecidos y vehículos, de existir rutas iguales siempre se elimina la de mayor costo. Es importante resaltar que en este trabajo se considera como objetivo minimizar el tráfico de carga, que es una función que relaciona la carga transportada con la distancia recorrida. HVRP-M es una simplificación de la problemática existente en la empresa CUPET, donde los recorridos y el montaje de carga de cada

vehículo se realiza manualmente. En trabajos futuros se pretende mejorar esta estrategia e incluir otras restricciones al problema para acercarse más a los requerimientos de la empresa.

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## Análisis experimental con metaheurísticas

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**Abstract**

La complejidad de los problemas actuales, así como las limitaciones en cuanto a tiempo y memoria para resolverlos, han hecho que el uso de las metaheurísticas cobre gran relevancia en dominios como la investigación de operaciones y las ciencias de la computación. La meta general en gran cantidad trabajos de investigación sobre optimización es la evaluación de una propuesta de algoritmo, planteando como hipótesis que este funcione mejor, en algún sentido, que un método existente. Un análisis experimental usualmente consiste en aplicar el algoritmo desarrollado a un conjunto de problemas y comparar la calidad de las soluciones obtenidas y los recursos que se han necesitado (en general tiempo de cómputo)[4]. En el presente trabajo se realiza un estudio experimental para mostrar cuál es el efecto que tienen algunas decisiones que se deben tomar para evaluar metaheurísticas. Se presentan varios escenarios donde se pone de manifiesto la importancia de seleccionar instancias representativas del problema. La dificultad de las instancias en problemas NP completos generalmente está asociada con una transición de fase [2], donde se encuentran las instancias más difíciles de resolver. Además el comportamiento del algoritmo puede variar en la resolución de instancias con diferente tamao, definiendo el tamao como la cantidad de variables que contiene. También se aborda la importancia de la elección de medidas de rendimiento que permitan informar de forma correcta los resultados, que incluyan tanto la calidad de las soluciones como el esfuerzo computacional necesario para alcanzarlas y la influencia que tiene la cantidad de ejecuciones independientes realizadas en el análisis de precisión de los algoritmos. Un

aspecto clave a la hora de evaluar metaheurísticas es la velocidad de computación con la que encuentra las soluciones. En este campo el esfuerzo computacional es medido típicamente por el número de evaluaciones que realizan y el tiempo de ejecución. Se analiza para el caso de los algoritmos paralelos la métrica speedup [1], [5], y algunas otras que dependen de su resultado. Los algoritmos utilizados en los experimentos son Algoritmo Genético y Enfriamiento Simulado, prestando especial atencin a sus versiones paralelas, para resolver problemas sobre la biblioteca Mallba [3].

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## A fair Hermite quadratic $\mathcal{A}$-spline scheme

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Jorge Estrada Sarlabous, ICIMAF
Wilfredo Morales Lezca, Universidad de la Habana

**Abstract**

In this work we present a new scheme for Hermite interpolation of a given set of planar points with a conic $\mathcal{A}$-spline curve.

The conic $\mathcal{A}$-spline is represented as a piecewise rational quadratic Bezier spline curve, which depends on local tension parameters in order to control the shape of each section.

We define a fairness functional in terms of the arc length and the bending energy of the curve. The values of the tension parameters minimizing this energy functional determine the fairest curve. Applying a subdivision scheme for conic $\mathcal{A}$-splines introduced in [2], we obtain good approximations of the functional and its derivatives, which are used for an efficient numerical computation of its minimum value.

The proposed Hermite quadratic $\mathcal{A}$-spline scheme is affine invariant, reproduces arcs of circles and satisfies the fairness requirements listed in [4].

The fair Hermite quadratic $\mathcal{A}$-spline scheme has been implemented in Matlab and a gallery of results is shown.

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# Asymptotics of minimal discrete periodic energy problems


D. P. Hardin, Vanderbilt University
E. B. Saff, Vanderbilt University
B. Z. Simanek, Baylor University
Y. Su, Vanderbilt University



**Abstract**

Let $L$ be a $d$-dimensional lattice in $R^d$. For a parameter $s > 0$, we consider the asymptotics of $N$ point configurations minimizing the $L$-periodic Riesz $s$-energy as the number of points $N$ goes to infinity. In particular, we focus on the case $0 < s < d$ of long-range potentials where we establish [1] that the minimal energy $E_s(L, N)$ is of the form $E_s(L, N) = C_0N^2 + C_1N^{1+s/d} + o(N^{1+s/d})$ as $N \to \infty$. The constants $C_0$ and $C_1$ depend only on $s$, $d$, and covolume of the lattice $L$.


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## Optimization in water distribution systems

Lisa C. Hegerhorst, Leibniz Universität
Marc C. Steinbach, Leibniz Universität

**Abstract**

In this talk we give an overview about optimization in water distribution systems. We consider the mathematical modelling of such systems, which leads to large scale MINLP. Furthermore we present former approaches to tackle these problems and discuss difficulties as well as unresolved issues.

## Subdivision based snakes for contour detection

V. Hernández Mederos, ICIMAF
R. Díaz Fuentes, ICIMAF
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**Abstract**

Active contours or snakes were introduced by Kass et al. in [5] as a curve that slithers from an initial position to the contour of a bounded region in a digital image. The evolution of the snake is guided by the minimization of several energies that measure the proximity between the snake and the contour and also control some desirable properties of the final curve, such as the smoothness, the interpolation of distinguished points, etc. Most of the snakes reported in the literature may be classified in three categories: point, geodesic and parametric snakes. Parametric snakes are smooth curves described by a small set of coefficients in a suitable basis of functions. The algorithms to compute them minimizing some energies are faster than the corresponding algorithms for point and geodesic snakes.

In this paper we use a particular class of parametric snakes generated from a subdivision scheme. Since subdivision schemes provide a hierarchical representation of smooth contours, they are very popular in geometric modeling applications. In the context of image segmentation most of the snakes based on subdivision schemes are those producing B-spline type curves [4], [1]. The snake proposed in this work is generated by the classical four point subdivision scheme [2], which has been also used in [3], [5].

In our approach, the control points of the snake are computed minimizing a combined energy which depends on gradient and region image energies and also on the curvilinear parametrization of the subdivision curve. We introduce a new region energy inspired in the

results of [1], which pushes the snake towards the boundary of the interest region, when the initial approximation is far from the contour. The proposed method profits from the hierarchical nature of subdivision curves, since the unknowns of the optimization process are the few coefficients of the curve in the coarse representation and, at the same time, good approximations of the energies and their derivatives are obtained from the fine representation. The algorithm was implemented in Matlab and the result for synthetic and real images are shown.

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# Optimization problems in the stability analysis of uncertain time-varying systems

Diederich Hinrichsen, Universität Bremen

**Abstract**

We consider a network of $N$ time-varying linear subsystems described by

$$\mathbf{\Sigma}_i : \dot{x}^i(t) = A_i(t)x^i(t) + B_i(t)u^i(t), \quad y^i(t) = C_i(t)x^i(t), \quad t \in \mathbb{R}_+ := \{t \in \mathbb{R}; t \geq 0\},$$

where $A_i(\cdot) \in L^\infty_{loc}(\mathbb{R}_+; \mathbb{K}^{n_i \times n_i})$, $B_i(\cdot) \in L^\infty(\mathbb{R}_+; \mathbb{K}^{n_i \times \ell_i})$, $C_i(\cdot) \in L^\infty(\mathbb{R}_+; \mathbb{K}^{q_i \times n_i})$ are given, $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$. The interconnection structure between these subsystems is described by an interconnection matrix $E = (e_{ij}) \in \{0,1\}^{N \times N}$: The output $y^j$ of $\mathbf{\Sigma}_j$ can influence the input $u^i$ of $\mathbf{\Sigma}_i$ if and only if $e_{ij} = 1$.
The corresponding vector space of uncertain coupling matrices of structure $E$ is given by:

$$\mathbf{\Delta}_E := \{\Delta = (\Delta_{ij});\ \Delta_{ij} \in \mathbb{K}^{\ell_i \times q_j},\ \Delta_{ij} = 0 \text{ if } e_{ij} = 0\}.$$

With every time-varying coupling matrix $\Delta(\cdot) = (\Delta_{ij}(\cdot)) : \mathbb{R}_+ \to \mathbf{\Delta}_E$ we associate the interconnected system

$$\dot{x}_i(t) = A_i(t)x_i(t) + B_i(t)\sum_{j=1}^{N}\Delta_{ij}(t)y_j(t) = \\ = A_i(t)x_i(t) + B_i(t)\sum_{j=1}^{N}\Delta_{ij}(t)C_j(t)x_j(t), \quad i \in \underline{N}. \quad (5)$$

We assume that the subsystems $\mathbf{\Sigma}_i$ are uniformly exponentially stable, i.e. their transition matrices $\Phi_i(t,s)$ satisfy $\|\Phi_i(t,s)\| \leq M_i e^{-\omega_i(t-s)}$ for all $t \geq s \geq 0$ where $M_i$, $\omega_i$ are positive constants. The time-varying block matrices $\Delta(\cdot) = (\Delta_{ij}(\cdot)) \in L^\infty(\mathbb{R}_+; \mathbf{\Delta}_E)$ model uncertain time-varying couplings between the subsystems (via $(\Delta_{ij}(\cdot), i \neq j)$ and time-varying parametric uncertainties in each subsystem (via $\Delta_{ii}(\cdot)$).
The central problem is: *For which bounds on the $L^\infty$-norms of the uncertain couplings $\Delta(\cdot) \in L^\infty(\mathbb{R}_+; \mathbf{\Delta}_E)$ can we guarantee the uniform exponential stability of the interconnected system $\mathbf{\Sigma}_{\Delta(\cdot)}$ described by (5).* The supremal value of these bounds is called the *stability radius* of the uncertain interconnected system and is denoted by $r(A, B, C, E)$. To determine $r(A, B, C, E)$ is a difficult optimization problem, see [5,§5.6]. Some results are available for the special case $N = 1$ (uncertain single time-varying system, see [1], [2]) and for networks of *time-invariant* subsystems $\mathbf{\Sigma}_i$, see [4]. In my talk I will give a brief survey about these results and present some new ones for interconnected *time-varying* systems. The results are obtained by a mix of methods from operator theory, Perron Frobenius theory and the theory of time-varying differential Riccati equations.

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# Ekeland's variational principle and its application to equlibrium problems

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S. Plubtieng, Naresuan University
A. Farajzadeh, Razi University

**Abstract**

In this study, we obtain a generalization of vectorial form of Ekeland's variational principle for set valued mapping in Banach space. As an application of our work, we provide some existence results for equilibrium problems in complete and noncomplete spaces.

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# A note on the existence of certain nowhere Lipschitz functions

Miguel Antonio Jiménez Pozo, Benemérita Universidad Autónoma de Puebla
Jorge Bustamente González, Benemérita Universidad Autónoma de Puebla

**Abstract**

For every $0 < \beta < \alpha < 1$, the following inclusions hold strictly,

$$lip_1 = Const \subsetneq C' \subsetneq D \subsetneq Lip_1 \subsetneq lip_\alpha \subsetneq Lip_\alpha \subsetneq lip_\beta \subsetneq C = lip_0 \subsetneq B = Lip_0,$$

where the chain of symbols denote the traditional linear spaces of real valued $2\pi-$periodic constant, differentiable, Lipschitz, continuous, and bounded functions, respectively. As known from Weierstrass example, there exists a function $f \in C$ that is nowhere differentiable. We refine this result by proving the existence of a function $f \in lip_\beta$ that is nowhere in $Lip_\alpha$.

# Analytic continuation of meromorphic functions defined on a three-sheeted Riemann surface via type I Hermite-Padé polynomials

Alexander Komlov, Steklov Mathematical Institute

**Abstract**

We consider meromorphic functions $f$ on a 3-sheeted Riemann surface that is constructed using the construction of so-called Nuttall condenser $(E, F)$ [1] ($E$ and $F$ are some compact sets). Let the function $f$ have no pole at $\infty$. Consider Hermite–Padé polynomials of the first kind for the triple $[1, f, f^2]$, that is, the polynomials $Q_{n,0}, Q_{n,1}, Q_{n,2}$ (of degree not greater than $n$) such that $Q_{n,0}(z) + Q_{n,1}(z)f(z) + Q_{n,2}(z)f^2(z) = O(z^{-2n-1})$, $z \to \infty$. We find the limiting distribution of zeroes of polynomials $Q_{n,j}$.

Using Nuttall's approach [2] it is possible to show that

$$\frac{Q_{n,1}}{Q_{n,2}}(z) \longrightarrow -\big\{f(z^{(1)}) + f(z^{(2)})\big\}, \quad \frac{Q_{n,0}}{Q_{n,2}}(z) \longrightarrow f(z^{(1)})f(z^{(2)}), \quad z \in \overline{\mathbb{C}} \setminus F.$$

We find the limiting distribution of interpolation points, that is, zeroes of the functions $\frac{Q_{n,1}}{Q_{n,2}}(z) + \big\{f(z^{(1)}) + f(z^{(2)})\big\}$ and $\frac{Q_{n,0}}{Q_{n,2}}(z) - f(z^{(1)})f(z^{(2)})$ (the same for both functions). Moreover, points of joint interpolation, that is, $\{z\colon Q_{n+1,j}(z)Q_{n,2}(z) - Q_{n,j}(z)Q_{n+1,2}(z) = 0\}$ for $j = 0, 1$, have the same limiting distribution.

These results are obtained jointly with N.G.Kruzhilin, R.V.Palvelev, and S.P.Suetin and announced in [3].

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# Multiplicative non-Hermitian perturbations of Hermitian matrices and classical random matrix ensembles

Rostyslav Kozhan, Uppsala University
Mikhail Tyaglov, Shanghai Jiao Tong University

**Abstract**

We fully classify possible eigenvalue configurations of rank-one multiplicative non-Hermitian perturbations

$$H(I + i\Gamma)$$

of Hermitian matrices $H$ ($\Gamma \geq 0$, $\operatorname{rank}\Gamma = 1$). We reduce such matrices to a tridiagonal Jacobi form, prove a Hermite-Biehler type theorem for the characteristic polynomial, and provide an application in random matrix theory.

# Ordered $D$-stability and its application to mathematical economics

Volha Kushel, Shanghai Jiao Tong University

**Abstract**

A matrix $\mathbf{A}$ is called *positive stable* (or just stable) if all its eigenvalues have positive real parts. A matrix $\mathbf{A}$ is called *$D$-stable* if $\mathbf{DA}$ is stable for every positive diagonal matrix $\mathbf{D}$. The concept of $D$-stability is widely used to describe economic processes (see [1], [2]). We study the following matrix property which is stronger than stability but weaker than $D$-stability. Given a permutation $\theta = (\theta(1), \ldots, \theta(n))$ of the set of indices $(1, \ldots, n)$, we call a matrix $\mathbf{A}$ *$D$-stable with respect to $\theta$* or $D_\theta$-stable if $\mathbf{DA}$ is positive stable for every positive diagonal matrix $\mathbf{D} = \operatorname{diag}\{d_{11}, \ldots, d_{nn}\}$, which satisfy the inequalities

$$d_{\theta(i)\theta(i)} \geq d_{\theta(i+1)\theta(i+1)}, \qquad i = 1, \ldots, n-1.$$

This property is also applied to study problems in mathematical economics.

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# On Nikishin systems with discrete components and weak asymptotics of multiple orthogonal polynomials

Guillermo López Lagomasino, Universidad Carlos III de Madrid
Alexandre I. Aptekarev, Keldysh Institute of Applied Mathematics
Andrei Martínez Finkelshtein, Universidad de Almería

**Abstract**

We consider multiple orthogonal polynomials with respect to Nikishin systems generated by two measures $(\sigma_1, \sigma_2)$ with unbounded supports ($\operatorname{supp}\sigma_1 \subseteq \mathbb{R}_+$, $\operatorname{supp}\sigma_2 \subseteq (-\infty, 0)$) and $\sigma_2$ discrete. A Nikishin type equilibrium problem in the presence of an external field acting on $\mathbb{R}_+$ and a constraint on $\mathbb{R}_-$ is stated and solved. The solution is used for deriving the contracted zero distribution of the associated multiple orthogonal polynomials.

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# Linear spectral transformations of a matrix of measures

Francisco Marcellán Español, Universidad Carlos III de Madrid

**Abstract**

In this talk, a summary of recent results ([1] and [2]) on linear spectral transformations of a matrix of measures supported on the real line will be presented.

Given a matrix polynomial $W(x)$, matrix bi-orthogonal polynomials with respect to the sesquilinear form $\langle P(x), Q(x)\rangle_W = \int P(x)W(x)d\mu(x)(Q(x))^T, P, Q \in \mathbf{R}^{p\times p}[x]$, where $\mu(x)$ is a matrix of Borel measures supported in some infinite subset of the real line, are considered. Connection formulas between the sequences of bi-orthogonal matrix polynomials with respect to $\langle\cdot,\cdot\rangle_W$ and matrix polynomials orthogonal with respect to $\mu(x)$ are deduced. In particular,

for the case of nonsingular leading coefficients of the perturbation matrix polynomial $W(x)$ a generalization of the Christoffel formula constructed in terms of the Jordan chains of $W(x)$ is deduced. For perturbations with a singular leading coefficient, several examples of matrix orthogonal polynomials by A. J. Durán et al are revisited. They appear as eigenfunctions of second order linear differential operators with matrix polynomials as coefficients.

The Geronimus spectral linear transformation, as an inverse of the above Christoffel transformation, will be also considered.

This is a joint work with G. Ariznabarreta and M. Mañas (Universidad Complutense de Madrid), J. C. García-Ardila (Universidad Carlos III de Madrid) and C. Álvarez-Fernández (Universidad Pontificia Comillas).

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## On rational functions without Froissart doublets

Ana Matos, Université de Lille 1
Bernd Beckermann, Université de Lille 1
George Labahn, University of Waterloo

**Abstract**

In this talk we consider the problem of working with rational functions in a numeric environment. A particular problem when modeling with such functions is the existence of Froissart doublets, where a zero is close to a pole. We discuss three different parameters which allow one to monitor the absence of Froissart doublets for a given general rational function. These include the euclidean condition number of an underlying Sylvester-type matrix, a parameter for determing coprimeness of two numerical polynomials and bounds on the spherical derivative. We show that our parameters sharpen those found in previous papers [1], [2].

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## On external fields created by fixed charges

Ramón Orive, Universidad de La Laguna

**Abstract**

This talk is devoted to the study of equilibrium problems in the real axis in the presence of external fields created by atomic measures. We are mainly concerned with the number of cuts (intervals) comprising the support of the equilibrium measure. Though we will deal mainly with the logarithmic potential setting, the case of general $s$–Riesz potentials, for $0 < s < 1$, will be also considered.

This talk is based on recent joint works with J. F. Sánchez Lara (Granada Univ., Spain), P. Dragnev (IPFW, Indiana, USA) and D. Benko (Univ. South Alabama, Mobile, USA).

## Curve and surface fitting by implicit polynomials: optimum degree finding and heuristic refinement

Rubén Interián, Fluminense Federal University
Juan Manuel Otero, Universidad de La Habana

**Abstract**

Finding an implicit polynomial that fits a set of observations $X$ is the goal of many investigations in recent years. However, most existing algorithms assume the knowledge of the degree of the implicit polynomial that best represents the points. This paper proposes an algorithm capable of finding the degree of the polynomial needed for the representation of the data set. For this purpose, a new distance measure between $X$ and the implicit polynomial is defined. The proposed algorithm is based on the idea of gradually increasing the degree, as long as there is an improvement in the smoothness of the solutions. In particular we consider the application of the proposed algorithm using lemniscates as implicit polynomials.

# Muckenhoupt inequality with three measures and applications to Sobolev orthogonal polynomials

Eduardo Colorado, Universidad Carlos III de Madrid
Domingo Pestana, Universidad Carlos III de Madrid
José M. Rodríguez, Universidad Carlos III de Madrid
Elena Romera, Universidad Carlos III de Madrid

**Abstract**

The classical Muckenhoupt inequality with two measures:

$$\left\| \int_0^x f(t)\,dt \right\|_{L^q((0,\infty),\mu)} \leq C \|f\|_{L^p((0,\infty),\nu)} \tag{6}$$

which holds for all measurable functions $f$ in $(0,\infty)$ iff

$$B := \sup_{r>0} \mu\left([r,\infty)\right)^{1/q} \left[ \int_0^r \left( \frac{d\nu}{dt} \right)^{-1/(p-1)} dt \right]^{(p-1)/p} < \infty,$$

appears in many contexts of mathematics, see for example (pp.40, [5]), where we find an equivalent condition for the estimate that some measures must hold, that is in connection with the condition for the classical $A_p$ weights, see for instance [2]. It is also related with the classical Hardy inequality, which is also known as an expression of the Heisenberg uncertainty principle, first formulated as a principle of quantum mechanics in 1927, see [3]. In harmonic analysis, estimates of different operators with respect to weights have been largely studied; In the classical book [1] we find a general presentation of the theory.

We generalize (6) to three measures under certain conditions. The field of application of our new Muckenhoupt inequality will be weighted Sobolev spaces, and, in particular, the multiplication operator (MO) defined by $Mf(z) = z\,f(z)$. Our interest in the MO arises from its relationship with the control of the zeros of Sobolev Orthogonal Polynomials (SOP). In the theory of SOP we don't have the usual three term recurrence relation for orthogonal polynomials in $L^2$ so, it is really difficult to find an explicit expression for the SOP of degree $n$. Hence, one of the central problems in the study of these polynomials is to determine its asymptotic behavior. In [4] it was shown how to obtain the $n$th root asymptotic of SOP if the zeros of these polynomials are contained in a compact set of the complex plane. Although the uniform bound of the zeros of orthogonal polynomials holds for every measure with compact support in the case without derivatives, it is an open problem to bound the zeros of SOP with respect to the Sobolev norm, but the boundedness of the zeros is a consequence of the boundedness of the MO in the completion of the linear space of polynomials with respect to the Sobolev norm; in fact, the zeros of the SOP are contained in the disk $\{z : |z| \leq 2\|M\|\}$ (see Theorem 2.1 [4]).

As a consequence of our Muckenhoupt inequality we prove a very simple characterization of the boundedness of MO and thus the boundedness of the zeros and the asymptotic behaviour of SOP for a large class of measures which includes the most usual examples in the literature.

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## Approximation algorithms for Ridge functions

Allan Pinkus, Technion

**Abstract**

In this talk we are motivated by the problem of approximating by Ridge Functions. We are given a normed linear space $X$, $r$ closed linear subspaces $\mathcal{M}^i$, $i = 1, \ldots, r$, and best approximation operators $P_i$ from $X$ to $\mathcal{M}^i$, $i = 1, \ldots, r$. Our goal is to construct and analyze algorithmic methods, using these $P_i$, of finding a best approximation from $\overline{\mathcal{M}}$, where

$$\mathcal{M} = \mathcal{M}^1 + \cdots + \mathcal{M}^r.$$

# Asymptotics of extremal polynomials with respect to Sobolev norms

Ana Portilla, St. Louis University
Yamilet Quintana, Universidad Simón Bolívar
José M. Rodríguez, Universidad Carlos III de Madrid
Eva Touris, Universidad Autónoma de Madrid

**Abstract**

In the context of Sobolev orthogonal polynomials there is no such a thing as the usual three term recurrence relation for orthogonal polynomials in $L^2$. Then, since finding an explicit expression for the extremal polynomial of degree $n$ becomes very complicated, an asymptotic estimate for the behavior of those extremal polynomials is especially interesting.

Let $\mathbb{P}$ be the space of polynomials with complex coefficients endowed with a non-diagonal Sobolev norm $\|\cdot\|_{W^{1,p}(V\mu)}$, where the matrix $V$ and the measure $\mu$ constitute a $p$-admissible pair for $1 \leq p < \infty$. In this work we establish the zero location and asymptotic behavior of extremal polynomials associated to $\|\cdot\|_{W^{1,p}(V\,\mu)}$, stating hypothesis on the matrix $V$ rather than on the diagonal matrix appearing in its unitary factorization.

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# Discrete Painlevé equations for recurrence coefficients of Laguerre-Hahn orthogonal polynomials of class one

Maria das Neves Rebocho, Universidade da Beira Interior, Universidade da Coimbra

Abstract

This talk is devoted to the study of recurrences for Laguerre-Hahn orthogonal polynomials of class one. It is shown for some families of such Laguerre-Hahn polynomials that the coefficients of the three term recurrence relation satisfy some forms of discrete Painlevé equations, namely, $dP_I$ and $dP_{IV}$.

This is based on joint work with G. Filipuk [1].

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# A reproducing kernel Hilbert discretization method for linear PDEs with nonlinear right-hand side

Eugenio M. Rocha, University of Aveiro

**Abstract**

The main problem of study of this work concerns to find a suitable discretization method to (numerically represent) the solution $u \in \mathcal{H}$, in a Hilbert space, of the equation

$$L[u(x)] = f(x, u(x)), \quad x \in \Omega \subset \mathbb{R}^N,$$

satisfying a given boundary condition (e.g. Dirichlet), where $L$ is a linear differential operator and $f$ is a nonlinear function with enough regularity. The method *Aveiro Discretization Method in Mathematics* (ADMM), introduced by Saitoh et al. (e.g. see [1]), can deal with problem above when $f$ is a function that do not depends on $u$. In fact, ADMM is a general method for solving by discretization, in a specific optimal sense and when *a priori* some data may be missing, a wide class of linear mathematical problems by using some key ideas of reproducing kernels and Tikhonov regularization theory. Here, we aim to extend the ADMM method to a more general situation where the nonlinearity may depend on $u$, see [2]. Then, we apply the scheme to find the (optimal) discretization solution of the problem

$$\vartheta \frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial^2 u}{\partial x^2} + \gamma u = u^3, \quad \text{on } \mathbb{R}^2,$$

for arbitrary coefficient functions $\vartheta, \alpha, \gamma$. Applications of the extension to PDEs involving nonlocal operators may be discussed, e.g. to Schrödinger-Poisson systems, beside others.

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## Hybrid nonautonomous SIR-model coming from a simple and reasonable government action police

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**Abstract**

Some virus diseases (e.g. Zika, Chikungunya or Dengue) are a strong public concern because of its rapid spread even after a wide government intervention, usually by controlling the disease vectors (e.g. killing mosquitoes). Mathematically, the common model to represent such behavior is the SIR model and its variants [2]. In this work, we show that if we consider a SIR model plus a simple and reasonable government action policy (action strategy), the complete model is a complex structure that no more is a ordinary differential equation (or even a differential inclusion). To understand such structure, we use techniques of Dynamical Systems (e.g. nonautonomous attractors, center manifolds, Bohl exponents), e.g. see [1], [4] y [5], Hybrid Logics (e.g. hybrid automata, transition systems), e.g. see [3], and First-Order Logics (e.g. delta-complete decision, delta-satisfiability, SMT), e.g. see [6]. Note that the complete model has trajectories that do not appear when studying the SIR model without a action strategy. Additionally, new stability results and 1-dimensional reduction equation are presented for the SIR model. Our mathematical approach, based on hybrid numerical analysis, give clues about the reason why the government action police may be major reason for the periodic behavior of Dengue. The numerical analysis is done using our package pHL of the computational sDL system [7].

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# Markov-type inequalities and duality in weighted Sobolev spaces

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**Abstract**

If $w$ is a weight on the real line, the inequality

$$\|P'\|_{L^2(w)} \leq Cn^{\alpha}\|P\|_{L^2(w)}, \tag{7}$$

for all polynomials $P$ of degree at most $n$, and some appropriate constants $C$ and $\alpha$ depending just on $w$, is called the Markov-type inequality for the weight $w$. In this work we study the following problem: Find the analogous of Markov-type inequalities in the setting of weighted Sobolev spaces. A partial answer of this problem was given in [5].

The first part of this talk is devoted to provide another partial solution of the above problem, which is based on the adequate use of classical (non-Sobolev) Markov-type inequalities (7) (see [1], [2] ,[7]), in the setting of weighted Sobolev spaces, when the considered weights are generalized classical weights.

In the second part we study some basic facts about Sobolev spaces with respect to measures: separability, reflexivity, uniform convexity and duality, which are not available in the current literature. These Sobolev spaces appear in a natural way and are a very useful tool when we study the asymptotic behavior of Sobolev orthogonal polynomials (see [3], [4] and [6]).

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## Redes complejas con aplicación a la epidemia de dengue

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**Abstract**

La conocida importancia que tiene construir modelos matemáticos apropiados para caracterizar la dinámica de epidemias con vistas a predecir y controlar el riesgo de expansión de enfermedades, nos impulsó a estudiar un enfoque más actual acerca del estudio del comportamiento de las epidemias. La teoría de redes complejas es unos de los cuerpos teóricos que ha servido como sustento a las nuevas propuestas de modelación epidémica y con el cual hemos experimentado en nuestro trabajo. Se da introducción a los conceptos sobre este tema, su relación con la teoría de grafos y se aplica en el caso de la epidemia de dengue. Para verificar los resultados obtenidos para el caso de la red compleja, se realizó un software y se compara con un sistema de ED's que describe la dinámica de la epidemia del dengue.

## Warm started active set solver for tree-structured QPs

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Marc C. Steinbach, Leibniz Universität
Jens Huebner, Leibniz Universität

**Abstract**

We consider SQP methods for large structured QPs where a specialized sparse solver is available for the KKT system in an active set QP solver. Our goal is a generic active set algorithm that employs any KKT solver in a slack relaxation of the QP to avoid a phase 1. As a concrete example we study NLPs on trees arising, e.g., in robust model predictive control. We discuss applicable warm-start schemes for the QP solution and the reusability of factorizations on the KKT level.

## Simulated tropism: A new metaheuristic for multimodal optimization problems

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Carlos Saez, Universidad de Artemisa

**Abstract**

In this paper the metaheuristic Simulated Tropism (ST) for solving multimodal optimization problems is proposed. The natural metaphor on which this algorithm is based is phototropism, a process in which a hormone called auxin plays an important role, concentrating itself on the opposite region to the one where the light falls. This leads to a greater cell proliferation in this region, which results in an inclination of the plant towards the light source.

ST works with a population of n points, where n is the dimension of the search space. These points represent cells in a transversal cut of the stem of a plant. The value of the objective function in each point represents the intensity of the light stimulus it receives. We simulate the growth of the plant trough a displacement of the cells in a certain direction. In order to simulate the function of auxin, an additional growth in the point with the worst evaluation of the objective function is considered.

ST doesnt deal with swarms of fishes or birds that can move independently. Here the cells represent part of a stem and therefore must maintain a certain organic unity. Therefore, a similarity parameter is used which forces the cells not to modify the distances between them abruptly.

Numerous well known test optimization problems have been solved by ST with excellent results. The algorithm has demonstrated to be capable of obtaining several global solutions.

# Optimal discrete measures for Riesz potentials

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**Abstract**

We investigate the maximal $N$-point polarization (Chebyshev) problem on a manifold $A$; namely, finding points $x_1, x_2, \ldots, x_N$ on $A$ that maximize the minimum on $A$ of the discrete Riesz $s$-potential $\sum_{i=1}^{N} \frac{1}{|x-x_i|^s}$ over all $N$-point multi-sets. For fixed $N$, in the limit as the Riesz parameter $s \to \infty$, the polarization problem becomes the best-covering problem on $A$. We are particularly interested in the large $N$ limit of such maximizing point configurations when $s > \dim(A)$, the so-called hypersingular case. Comparisons with minimal Riesz $s$-energy configurations will be discussed. Work is joint with S. Borodachov, D. Hardin, and A. Reznikov.

# Patrones fuertes de Turing-Hopf en el sistema de Lengyel-Epstein

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**Abstract**

Los modelos de reacción-difusión se han propuesto para estudiar la formación de patrones en una amplia variedad de fenómenos en la naturaleza [1]. Turing [2] demostró que bajo ciertas condiciones el término difusivo actúa como un mecanismo desestabilizante del estado estacionario de la parte reactiva del modelo conduciendo esto a la formación de patrones. El primer experimento que evidencia los llamados patrones de Turing fue observado por De Kepper [3],[4] en una reacción química de ácido de clorito-yoduro-malónico (reacción-CIMA ), posteriormente Lengyel-Epstein [5] estudiaron este resultado utilizando un sistema de reacción-difusión, el mismo ha sido extendido por diferentes autores [6], [7], [8] tomando en consideración interacciones de las inestabilidades Turing con bifurcaciones de Hopf para un espacio paramétrico dado. Motivado por el trabajo [9] retomamos el sistema de Lengyel-Epstein para investigar los patrones de Turing debido ahora a las oscilaciones del ciclo limite presente en la parte reactiva de modelo, este nuevo escenario es denominado patrones fuertes de Turing-Hopf.

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## On series representations for Apéry's constant

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**Abstract**

The purpose of this work, inspired by some results of Rhin and Viola [1] y [7], is to present a series representation for $\zeta\,(3)$, which only depends on one single integer parameter. In order to complete this, we will deduce a Hermite-Padé approximation problem using Sorokin's ideas [3], [4], [5] y [6]. Moreover, as a consequence we get a new recurrence relation linked to $\zeta(3)$ as well as a new continued fraction expansion [2] for $\zeta(3)$. Finally, the convergence rate of several series representations of $\zeta(3)$ are compared.

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## Analytic properties of Al-Salam-Carlitz I-Sobolev type orthogonal polynomials

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**Abstract**

In this contribution we consider the sequences of monic polynomials orthogonal with respect to the Sobolev-type inner product where $a < 0$, $N \in \mathbb{R}_+$, $\xi \notin [a, 1]$ and $\mathscr{D}_q$ denotes the $q$-derivative or the Euler-Jackson $q$-difference operator [1] defined by

$$(\mathscr{D}_q f)(x) := \begin{cases} \dfrac{f(qx) - f(x)}{(q-1)x} & \text{if } x \neq 0,\ q \neq 1, \\ f'(x) & \text{if } x = 0,\ q = 1. \end{cases}$$

We derive an explicit representation for these polynomials, as well as we deduce its hypergeometric representation. The ladder operators associated with these polynomials are obtained. The $q-$analogue of the holonomic equation is also given. Moreover, we present some results on the distribution of its zeros. Here, we analyze the behavior of its zeros as a function of the mass $N$. In particular, we obtain such a behavior when the mass $N$ tends to infinity as well as we characterize the exact values of $N$ such the smallest (respectively, the largest) zero of the studied polynomials is located outside $[a, 1]$. Finally, for these polynomials we derive a five-term recurrence relation.

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# Anisotropic approximation with shift-invariant subspaces

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**Abstract**

Consider the Lebesgue space $L^2(\mathbb{R}^d)$ of square summable functions with norm $\| \|_2$. A shift-invariant subspace $V$ is a closed subspace of $L^2(\mathbb{R}^d)$ such that if $f \in V$ and $k \in \mathbb{Z}^d$, then $f(\cdot - k) \in V$. Associated to a given dilation $A$ on $\mathbb{R}^d$, consider the operator $\mathcal{D}_A f(\cdot) = |\det(A)|^{1/2} f(A\,\cdot)$ ($f \in L^2(\mathbb{R}^d)$). The shift-invariant subspaces $V$, $H$ are said to be $A$-refinable and $A$-reducing, respectively, if $V \subseteq \mathcal{D}_A V$ and $H = \mathcal{D}_A H$. We give several conditions equivalent to the completeness property ($\bigcup_{j\in\mathbb{Z}} \mathcal{D}_A^j V$ is dense in $H$) of an $A$-refinable subspace $V$ (see [2]).

Let $P_V$ denote the orthogonal projection on $V$. We consider the anisotropic Sobolev space $W_A^{\alpha,2}$ given by the norm $\|f\|_{A,\alpha} = \|(1+\rho)^\alpha \widehat{f}\|_2$, where $\alpha \geq 0$, $\widehat{f}$ is the Fourier transform of $f$, and $\rho$ is a pseudo-norm for $A^*$, conjugate of $A$. We say that $V$ provides $A$-approximation order $\alpha > 0$ if there exists $C > 0$ such that

$$\|f - P_{\mathcal{D}_A^j V} f\|_2 \leq C\,|\det(A)|^{-j\alpha/d}\,\|f\|_{A,\alpha} \qquad \forall\, f \in W_A^{\alpha,2},\, j \in \mathbb{Z}\,.$$

Also, we say that $V$ provides $A$-density order $\alpha \geq 0$ if

$$|\det(A)|^{j\alpha/d}\,\|f - P_{\mathcal{D}_A^j V} f\|_2 \xrightarrow[j\to\infty]{} 0 \qquad \forall\, f \in W_A^{\alpha,2}\,.$$

We give a characterization of the anisotropic approximation and density orders of a shift-invariant subspace $V$ (see [1]).

These results generalize several others recently appeared in the literature. All the provided conditions focus on the local behaviour at the origin of the spectral function of $V$, making use of the previously introduced notions of anisotropic density point of a set and anisotropic approximate continuity point of a measurable function. Joint work with P. Cifuentes and A. San Antolín.

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# A distributed interior point method for multistage stochastic NLPs

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**Abstract**

Interior point methods are well-suited for solving multistage stochastic NLPs when an efficient algorithm for the huge structured KKT systems is available. This is the case for large scenario trees with a moderate number of variables per node: we present a distributed "tree-sparse" solution algorithm based on a static partitioning of the tree and featuring low memory and communication overheads. We also address structured quasi-Newton updates for the sparse Hessian as well as structured inertia corrections to address non-convexity or rank-deficiency of the KKT system. Computational results for benchmark problems from portfolio optimization and robust model predictive control demonstrate the performance of our approach.

# Energy bounds for antipodal codes on $\mathbb{S}^{n-1}$

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**Abstract**

Let $C \subset \mathbb{S}^{n-1}$ be antipodal spherical code (i.e. $C = -C$) with inner products in $\{-1\} \cup [-s, s]$, where $s \in (0, 1)$, $|C| = M$. For a given (extended real-valued) function $h : [-1, 1] \to [0, +\infty]$, we consider the *h-energy* (or the potential energy) of $C$ defined by

$$E(n, C; h) := \frac{1}{|C|} \sum_{x,y \in C, x \neq y} h(\langle x, y \rangle),$$

where $\langle x, y \rangle$ denotes the inner product of $x$ and $y$. The potential function $h$ is called $k$-*absolutely monotone* on $[-1, 1)$ if its derivatives $h^{(i)}(t)$, $i = 0, 1, \ldots, k$, are nonnegative for every $t \in [-1, 1)$.

An antipodal spherical code $C \subset \mathbb{S}^{n-1}$ is called $k$-A-universally antipodal if it has minimum possible $h$-energy among all antipodal codes on $\mathbb{S}^{n-1}$ of the same cardinality $M = |C|$.

In this work we are interested in the $k$-A-universally optimal antipodal codes for some small values of $k$. Typically, our codes have a few distinct inner products. We provide a

general linear programming bound on energy of antipodal codes and exhibit certain antipodal codes that are $k$-A-universally optimal for some small $k$.

**Theorem 1.** *Let $f(t) = \sum_{i=0}^{\deg(f)} f_i P_i^{(n)}(t)$ be a real polynomial such that*

(A1) *$f(t) \le h(t)$ for $t \in [-s, s]$, and*

(A2) *the Gegenbauer coefficients satisfy $f_i \ge 0$ for every even $i$.*

*Then*

$$E(n, C; h) \ge M f_0 - f(1) - f(-1) + h(-1)$$

*for every antipodal code $C \subset \mathbb{S}^{n-1}$ of cardinality $M = |C|$ and inner products in $\{-1\} \cup [-s, s]$.*

Moreover, we prove Lloyd's type theorem for these cases.

**Theorem 2.** *If an antipodal code $C \subset \mathbb{S}^{n-1}$ of cardinality $M = |C|$ and inner products $-1$, $\pm s$ is 4-universally optimal, then $(n, M, s) = (n, 4n, 1/\sqrt{2n-1})$ or $s = \sqrt{\frac{M-2n}{n(M-2)}}$ is rational and $\frac{M}{4} + \frac{M-4n}{4sn}$ is positive integer.*

**Theorem 3.** *If an antipodal code $C \subset \mathbb{S}^{n-1}$ of cardinality $M = |C|$ and inner products $-1$, $0$ and $\pm s$ is 6-A-universally optimal, then: $s = \sqrt{\frac{3M-2n(n+2)}{(n+2)(M-2n)}}$ is rational and $A_s(x) = \frac{(n+2)(M-2n)^2}{2n(3M-2n(n+2))}$, $X = \frac{(M-4n)(1+s)}{4ns} + \frac{(n-1)M(M^2-8nM+4n^2(n+2))}{4n(3M-2n(n+2))^2}$ and $Y = \frac{(n-1)(M-n(n+1))M^2}{n(3M-2n(n+2))^2}$ are nonnegative integers.*

Using Theorem 1 we prove that some codes are $k$-A–universally optimal for $k = 4$ or $k = 6$. For example, in terms of (dimension, cardinality, maximal inner product), the antipodal spherical codes with parameters $(n, 2n+2, 1/n)$, $(5, 20, 1/3)$, $(6, 32, 1/3)$ and $(7, 56, 1/3)$ are 4-A–universally optimal and $(4, 24, 1/2)$, $(6, 72, 1/2)$, $(7, 126, 1/2)$, $(22, 2816, 1/3)$ and $(23, 4600, 1/3)$ are 6-A–universally optimal.

## Relative asymptotics of orthogonal polynomials for perturbed measures

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**Abstract**

In the context of orthogonal polynomials in the plane we introduce the notion of a polynomially small (PS) perturbation of a measure. Namely, if $\mu_0 \ge \mu_1$ and $\{p_n(\mu_j, z)\}_{n=0}^{\infty}, j = 0, 1$, are associated orthonormal polynomial sequences, then $\mu_0$ a PS perturbation of $\mu_1$ if $\|p_n(\mu_1, \cdot)\|_{L_2(\mu_0 - \mu_1)} \to 0$, as $n \to \infty$. In such a case we establish relative asymptotic results for the two sequences of orthonormal polynomials. As an application, we study the asymptotic behaviour of the zeros of Bergman polynomials in archipelaga with lakes. This is a report of joint work with E.B. Saff.

# Heuristics algorithms from linear equilibrium constrained problems

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**Abstract**

Linear equilibrium constrained programming is a special class of optimization models with equilibrium constraints. Because of the complexity of the equilibrium condition it is replaced by necessary conditions, which leads to a complementarity constrained problem (MPCC). The set of feasible solutions in a MPCC is structured as a union of polyhedrons. Solving the MPCC problem would require the minimization of the objective function on each of these polyhedrons. The computation cost of this approach is unfeasible, thus, this work presents a new approach where heuristic algorithms such as Hill Climbing and Simulated Annealing are used to search for good solutions on the polyhedrons space. A new benchmark for linear equilibrium constrained optimization is introduced. The computational results achieved by the proposed heuristics on the proposed benchmark are presented.

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# Nonlinear optimization in gas networks for storage of electric energy

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**Abstract**

The need for storage for highly volatile generated renewable energy can be addressed by electric compressor stations storing energy in terms of pressure increase. We present a transient optimization model that incorporates gas dynamics and technical network elements. Direct discretization leads to an NLP that is solved by interior point methods. We present results for simple pipelines as well as an analysis on the structure of the problem.

# Estudio de singularidades en transitorios de presión

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**Abstract**

La ruptura súbita en los sistemas de distribución de agua provoca gran pérdida de este recurso natural, causando que se interrumpa el abastecimiento además del daño ocasionado en calles y edificaciones, debido al flujo incontrolado del agua. Por este motivo para los acueductos es muy necesario realizar una adecuada detección y localización de este tipo de evento. En este trabajo se propone un novedoso algoritmo para detectar y localizar eventos de ruptura súbita a través de la transformada wavelet. Este algoritmo consiste en realizar la estimación de las irregularidades locales de la función que representa una señal a través de sus coeficientes de detalle, comparándolos con un umbral de detección. Este método fue validado en el laboratorio del Centro de Investigaciones Hidrulicas (CIH) del Instituto Superior Politécnico José Antonio Echevarría (ISPJAE), donde se realizó el experimento de validación en una tubería que presenta una longitud de 26 m. Los resultados obtenidos muestran la gran veracidad del método pues se observa claramente como existe una alta repetitividad en la detección y localización de la señal, disminuyendo el costo computacional.

## Asymptotic zero distribution of Jacobi-Piñeiro and multiple Laguerre polynomials

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**Abstract**

We give the asymptotic distribution of the zeros of Jacobi-Piñeiro polynomials and multiple Laguerre polynomials of the first kind. We use the nearest neighbor recurrence relations for these polynomials and a recent result on the ratio asymptotics of multiple orthogonal polynomials. We show how these asymptotic zero distributions are related to the Fuss-Catalan distribution.

## Modified logarithmic potential theory and applications

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**Abstract**

Motivated by particular biorthogonal ensembles in random matrix theory, we study weighted potential theory for the modified kernel

$$\log \frac{1}{|x-y||f(x)-f(y)|w(x)w(y)},$$

where $x, y \in K$, a closed subset of $\mathbb{C}$, $f : K \to \mathbb{C}$ is continuous, and $w$ is a weight. We consider the problem of minimizing the corresponding weighted energy and describe analogs of classical results, like the Frostman inequalities and asymptotics of the $n$-th diameters. We also obtain a Bernstein-Walsh inequality and a Bernstein-Markov property for functions of the form $p(g(z))q(f(z))$ where $p, q$ are polynomials and $f, g$ are holomorphic in a neighborhood of $K$. Probabilistic results such as large deviation principles can be derived.

This is a joint work with Thomas Bloom (Toronto), Norman Levenberg (Bloomington), and Vilmos Totik (Tampa and Szeged).

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# A MINLP model to optimize energy consumption in railway rapid transit networks

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**Abstract**

With reduced environmental impact becoming an increasingly important benefit of the rail transportation mode, improvement in efficiency and reduction in energy consumption has become a major concern to rail transit operators. In particular, this is so when designing railway timetables where some efforts have been made in order to incorporate energy consumption in the determination of the most convenient departure and arrival times of trains, considering typically one single train. There are also some recent attempts focused in the analysis of a complete line. However, some aspects like the effect of passengers' load and the topology of the line have been neglected.

Here a methodology is proposed to optimize energy consumption in rapid transit railway networks by means of a mathematical integer non linear programming (MINLP) model. In doing so, a number of relevant aspects are considered including e.g. slopes and curvatures in each track of the network, the aforementioned effect of passengers' load in conditions of fully attended demand or speed limits and other limitations imposed for security reasons. As illustration, some particular examples are considered.

# Convolution and product theorems for the special affine Fourier transform

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**Abstract**

The Special Affine Fourier Transform or the SAFT, which was introduced in 1994 by S. Abe and J.T. Sheridan [1], generalizes a number of well known unitary transformations, as well as, signal processing and optics related mathematical operations. The SAFT depends on 6 paramters $(a, b, c, d; p, q)$ with $ad - bc = 1$. It includes the fraction Fourier transform as a special case in which $a = \cos\theta, b = \sin\theta, c = -\sin\theta, d = \cos\theta, p = 0 = q$. The fractional Fourier transform plays an important role in signal processing and optics [2]. Unlike the Fourier transform, the SAFT does not work well with the standard convolution operation.

Recently, Q. Xiang and K. Y. Qin [3] introduced a new convolution operation that is more suitable for the SAFT and by which the SAFT of the convolution of two functions is the product of their SAFTs and a phase factor. However, their convolution structure does not work well with the inverse transform in sofar as the inverse transform of the product of two functions is not equal to the convolution of the transforms.

In this article we introduce a new convolution operation that works well with both the SAFT and its inverse leading to an analogue of the convolution and product formulas for the Fourier transform. Furthermore, we introduce a second convolution operation that leads to the elimination of the phase factor in the convolution formula obtained in [3].

We conclude that our construction of the convolution structure for the SAFT domain establishes the SAFT duality principle, that is, convolution in one domain amounts to multiplication in the transform domain and vice–versa. Our results can be used to develop the semi–discrete convolution structure for sampling and approximation theory linked with the Special Affine Fourier Transform.

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# Local Map

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