OrthoQuad 2014

International Symposium on Orthogonality, Quadrature and Related Topics

In memory of Pablo González Vera (1955–2012)

Book of Abstracts

Puerto de la Cruz, Tenerife, Spain **January 20–24, 2014**

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International Symposium on Orthogonality, Quadrature and Related Topics

This is an international conference in memory of Prof. Dr. Pablo González Vera, Professor of Applied Mathematics at University of La Laguna (Canary Islands), who passed away on July 11, 2012.

Prof. González Vera was a recognized specialist in Approximation Theory, Orthogonal Polynomials and Quadrature Formulae, and was author of more than one hundred fifty papers published in prestigious international journals in the area of Applied Mathematics. He also wrote several books, among which specially stands "Orthogonal Rational Functions", published by Cambridge University Press, and written in collaboration with Adhemar Bultheel (KU Leuven, Belgium), Erik Hendriksen (Netherlands) and Olav Njåstad (Univ. of Trondheim).

In this sense, the main topics of the conference will be Orthogonality and Quadrature, but other topics in Approximation Theory, Special Functions and related issues are also within the scope of the conference.

Contents

Pablo González Vera. Biographical notes

Pablo González Vera was born in Vallehermoso (La Gomera, Canary Islands) in January 25, 1955. He studied Mathematics at University of La Laguna (1974-79), and began teaching at this university in 1980. He started his research in Rational Interpolation, in collaboration with Prof. L. Casasús (now in Universidad Politécnica de Madrid) and defended his Doctoral Dissertation on Two–point Padé Approximation in 1985. That was the germ of a research group on Approximation Theory in University of La Laguna.

In the following years, he shared his interest in Rational Interpolation with another closely related topic, the Quadrature Formulae (Numerical Integration). From 1991 on, seven Doctoral Thesis on Orthogonality and/or Quadrature were supervised by Prof. González Vera, including the most recent one (by Francisco Perdomo), which was defended shortly after his passing away.

During his career, Prof. González Vera was author of more than one hundred fifty papers published in prestigious international journals in the area of Applied Mathematics. He also wrote several books, among which specially stands "Orthogonal Rational Functions", published by Cambridge University Press, and written in collaboration with Adhemar Bultheel (KU Leuven, Belgium), Erik Hendriksen (Netherlands) and Olav Njåstad (Univ. of Trondheim). This book, nowadays a main reference, was the result of an intensive work in common by the authors, whose team is also known colloquially as "the gang of the four".

Pablo died on July 11, 2012 at the age of 57. He was not only a well recognized mathematician but a remarkable human being. He was extremely modest, friendly, concerned, and warm with all that surrounded him. No wonder he has left a large number of friends and colleagues in Spain and abroad who miss him every day and mourn his untimely loss.

Figure 1: The gang of the four

Organizing and Scientific Committee

The Organizing Committee:

- Ramón Orive (Chair), Univ. de La Laguna, Spain
- Matías Camacho, Univ. de La Laguna, Spain
- Ruymán Cruz-Barroso, Univ. de La Laguna, Spain
- Carlos Díaz, Univ. de La Laguna, Spain
- C. González, Univ. de La Laguna, Spain
- Mateo Jiménez, Univ. de La Laguna, Spain
- Francisco J. Perdomo Pío, Univ. de La Laguna, Spain
- Fernando Pérez, Univ. de La Laguna, Spain
- Héctor Pijeira, Univ. Carlos III de Madrid, Spain
- J. C. Santos, Univ. de La Laguna, Spain
- Juan J. Trujillo, Univ. de La Laguna, Spain

The Scientific Committee:

- Adhemar Bultheel (Chair), KU Leuven, Belgium
- Guillermo L. Lagomasino, Univ. Carlos III de Madrid, Spain
- Francisco Marcellán, Univ. Carlos III de Madrid, Spain
- Olav Njastad, Univ. Trondheim, Norway
- Ramón Orive, Univ. de La Laguna, Spain

Meeting inscription and other information

The secretariat will be open from Monday 20 to Wednesday 22, from 8:30 to 9:30 A.M.

Other information:

- All morning conferences begin at 9:30AM.
- The Opening Ceremony and Welcome Cocktail will be in Hotel Beatriz on Monday 20 at 19:30PM.
- There will be an excursion to Mount Teide and a visit to the Astrophysics Observatory on Wednesday 22 at 12:00.
- The official dinner will be in *Restaurante "La Gañanía"* (Puerto de La Cruz) on Thursday 23 at 21:00 PM.

OrthoQuad 2014 Time–Table

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<tr><td>10:30-11:00</td><td colspan="4" rowspan="2">W. Van Assche</td><td colspan="4" rowspan="2">E. A. Rakhmanov</td><td rowspan="2">B. de la Calle</td><td colspan="3" rowspan="2">F. Perdomo</td><td>M. Redivo</td><td>Aidarous</td></tr>
<tr><td>11:00-11:30</td><td>A. Bultheel</td><td>S. Omar</td></tr>
<tr><td>11:30-12:00</td><td colspan="13">Poster Session and Coffee Break</td></tr>
<tr><td>12:00-12:30</td><td>Li. Garza</td><td>M. Castro</td><td>Khozan</td><td>Lu. Garza</td><td rowspan="4">Bus excursion to Las Cañadas, including a visit to Mt. Teide <u>Astrophysics Observatory</u></td><td>A. Kuijlaars</td><td>(†)</td><td rowspan="3">Open Problems Session</td></tr>
<tr><td>12:30-13:00</td><td>Lassarov</td><td>A. Martínez</td><td>Ismailov</td><td>Borrego</td><td>E. Medina</td><td>(†)</td></tr>
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<tr><td>18:00-18:30</td><td>S. Medina</td><td>Pejcev</td><td>Slimane</td><td>J. Illán</td><td>M. Piñar</td><td>(†)</td></tr>
</table>

- Official dinner of the Conference: Thursday 23, 21:00, at Restaurant <u>"La Gañanía"</u> (Puerto de la Cruz)

Abstracts

Plenary Talks

How big the orthonormal polynomial from the Steklov class can be?

Alexander Aptekarev
Institute of Applied Mathematics, Russia

Abstract

The famous problem of Steklov is to find bounds for the polynomial sequences, which are orthonormal with respect to the strictly positive weight. In 1921 V. A. Steklov made a conjecture that such a sequence of polynomials is bounded on the support of the orthogonality measure. In 1979 E. A. Rakhmanov disproved this conjecture constructing a weight from the Steklov class, for which a subsequence of the polynomials demonstrates a logarithmic growth at one point of the support. Then a natural question have arisen: how fast this growth could be?

Let S_δ be the space of measures σ on the unit circle, such that $\sigma'(\theta) > \delta > 0$ at every Lebesgue point, and let $\phi_n(z)$ be the orthonormal polynomials with respect to $\sigma \in S_\delta$. In our talk we consider the following variational problem. Fix $n \in \mathbb{N}$ and $\delta > 0$. Let

$$M_n = \sup_{\sigma \in S_\delta} \|\phi_n\|_{L^\infty(\mathbb{T})} = \sup_{\sigma \in S_\delta} |\phi_n(1)|.$$

Elementary considerations yield $M_n \lesssim \sqrt{n}$. Rakhmanov has proved in 1981, that $M_n \gtrsim \sqrt{n}/(\ln n)^{\frac{3}{2}}$.

The main result of our joint work with S.A. Denisov and D.N. Tulyakov is $M_n \gtrsim \sqrt{n}$. I.e. the elementary upper estimate is sharp.

Algebraic properties of robust Padé approximants

Bernhard Beckermann
Laboratoire Painlevé, UFR Mathématiques, Université Lille 1, France

Abstract

It has been conjectured [2] that recently introduced so-called robust Padé approximants computed through SVD techniques do not have so-called spurious poles [3], that is, poles with a close-by zero or poles with small residuals. Such a result would have a major impact on the convergence theory of Padé approximants since it is known that convergence in capacity plus absence of poles in some domain D implies locally uniform convergence in D.

Following [1], we prove in the present talk the conjecture for the subclass of so-called well-conditioned Padé approximants, and discuss related questions. It turns out that it is not sufficient to discuss only linear algebra properties of the underlying rectangular Toeplitz matrix, since in our results other matrices like Sylvester matrices also occur. This type of matrices have been used before in numerical greatest common divisor computations.

Joint work with Ana C. Matos (Lille).

References:

[1] B. Beckermann and A.C. Matos,*Algebraic properties of robust Padé approximants*. Manuscript (2013).

[2] P. Gonnet, S. Güttel and L. N. Trefethen, *Robust Padé approximation via SVD,* SIAM Review, 55 (2013), pp. 101-117.

[3] H. Stahl, *Spurious poles in Padé approximation*, J. Comp. Appl. Math., 99 (1998), 511-527.

Around Padé-type approximation and rational interpolation

Claude Brezinski
Université Lille , France

Abstract

Three ideas will be presented in this talk

1. In Padé-type approximants, the denominator can be arbitrarily chosen. We will show how to choose it so that, in addition, these approximants also interpolate the function to be approximated.

2. In barycentric rational interpolation, the weights of the interpolants can be arbitrarily chosen. We will how how to choose them so that, in addition, they also satisfy a Padé-type approximation property.

3. We will show how to write Padé approximants under a barycentric form.

Pablo González-Vera, a quadrature of his work

Adhemar Bultheel
Department of Computer Science, KU Leuven, Belgium.

Abstract

In this talk I will try to estimate the breadth and width of Pablo's mathematical work. Measuring the influence he had, and still has, on the work of all the people who have known him professionally. That ranges from the two-point Padé approximation that he started with, but that quickly came to blossom in many papers on rational approximation with many more points of interpolation (countably many). His favored application of these was the design of numerical quadrature formulas. This was mainly developed together with his seven PhD students in the group in La Laguna and colleagues from abroad. He was rarely the sole author of a paper showing his skill as a team player and an excellent team leader.

It is a difficult task to do this in just one lecture. So like quadrature is finding a square with the same area as a more amorphous region, I will only be constructing approximations that may be about exact for certain subsections but it will be largely an approximate recollection valid within rounding errors caused by observations done with finite precision and finite memory storage.

There are of course many other aspects of Pablo as an administrator, a sports enthousiast, a musician, a person, a husband, a father, a friend. It was impossible to collaborate with him and not instantly be charmed by his warm personality. His colleagues and students were friends by definition. However I will avoid this emotional quicksand and mainly stick to the mathematics in this lecture.

Matrix methods for quadrature formulas on the unit circle

María José Cantero
Universidad de Zaragoza, Spain

Abstract

In this talk we present some results concerning the computation of quadrature formulas on the unite circle. The recurrence relation satisfied by the orthogonal Laurent polynomials with respect to a measure defined on the unit circle, gives rise to a certain unitary five-diagonal matrix. This matrix can be used to compute the nodes and the weights of Szegö's quadrature formulas on the unit circle. On the other hand, these quadrature formulas can be computed alternatively using Hesenberg matrices.

Orthogonal polynomials are a particular case of orthogonal rational functions with prescribed poles. Szegö's quadrature formulas can be generalized to orthogonal rational functions. A way to calculate the nodes and the weights to the rational Szegö's quadrature formulas is using matrix representations for orthogonal rational functions with prescribed poles.

In both cases we make a comparative study using different matrix representations to compute such quadrature formulas. We illustrate the preceding results with some numerical examples .

The results presented in this talk are the fruit of joint works with Pablo González-Vera and one of his closest collaborators, Adhemar Bultheel.

Multiple Orthogonal Polynomials on the Unit Circle

Ruymán Cruz Barroso
Department of Mathematical Analysis, La Laguna University

Abstract

Multiple Orthogonal Polynomials on the Unit Circle (MOPUC) were introduced and studied by Judith Mínguez and Walter Van Assche in [1], and surprisingly, this is so far the only reference on this topic.

The purpose of this talk is to deepen the analysis of the properties of MOPUC, concerning mainly in the normality, their effective computation from recurrence relations and a Riemann-Hilbert analysis. Multiple Verblunsky coefficients will be also introduced and some open questions will be presented.

This is a joint work with C. Díaz Mendoza and R. Orive.

References:

[1] J. Mínguez Ceniceros and W. Van Assche, *Multiple Orthogonal Polynomials on the Unit Circle*. Constr. Approx. 28 (2008) 173–197.

There's something about approximation beyond extremality.

Bernardo De la Calle
Universidad Politécnica de Madrid, Spain

Abstract

Classical results on approximation of analytic functions by Taylor series or row sequences of Padé approximants have been extended over the last decades to other type of approximations by means of potential theory, with equilibrium measures and extremal approximants playing a major role.

In this lecture we will show how some of the above results can be extended to general classes of interpolatory (non-extremal) approximants as long as the information given by the table of interpolation points can be properly plugged into the formulation of the problem. Namely, we will be concerned by the characterization of the region of analytic (or meromorphic) continuation of a function in terms of the geometric rate of convergence of its approximants on a given compact set and by the extension of the classical Jentzsch-Szegö theorem on zeros of Taylor polynomials.

Phase transitions and equilibrium measures in random matrix models

Andrei Martínez-Finkelshtein
Universidad de Almería, Spain

Abstract

We are interested in the so-called phase transitions in the Hermitian random matrix models with a polynomial potential. Or, in a language more familiar to approximators, we study families of equilibrium measures on the real line in a polynomial external field. The total mass of the measure is considered as the main parameter, which may be interpreted also either as temperature or time. By phase transitions we understand the loss of analyticity of the equilibrium energy.

Our main tools are differentiation formulas with respect to the parameters of the problem, and a representation of the equilibrium potential in terms of a hyperelliptic integral. This allows to find a dynamical system that describes the evolution of families of equilibrium measures. On this basis we are able to systematically derive results on phase transitions, such as the local behavior of the system at all kinds of phase transitions. We discuss in depth the case of the quartic external field.

This is a joint work with R. Orive, and E. A. Rakhmanov.

A saga of canary Approximators: The legacy of Pablo

Francisco Perdomo Pío
Universidad de La Laguna, Spain

Abstract

Along his very fruitful career, Pablo González Vera was building a nice and quite large family of canary approximators and numerical integrators: his academic "sons". In the first part of this talk, a brief overview of the progressive growth of this family will be made. In the second part, some of the contributions of the youngest son of Pablo (that is, myself) will be revised

Zero distribution of Hermite–Padé polynomials

Evguenii A. Rakhmanov
Univerity South Florida in Tampa, USA

Abstract

The lecture will be devoted to a review of some old and new conjectures and results related to zero distribution (weak asymptotics) of Hermite–Padé polynomials.

Orthogonal polynomials for Minkowski's question mark function.

Walter Van Assche
KU Leuven

Abstract

Hermann Minkowski introduced a function in 1904 which maps quadratic irrational numbers to rational numbers and this function is now known as Minkowski's question mark function since Minkowski used the notation $?(x)$. This function turns out to be a monotone increasing and continuous function on $[0,1]$ with $?(0) = 0$ and $?(1) = 1$ which is singular. Hence it defines a singular continuous measure q on $[0,1]$ and one can show that the support of this measure is $[0,1]$. The question mark function is also known as the slippery devil's staircase. There are several ways to define the question mark function or the corresponding measure: one can use the continued fraction expansion of real numbers in $[0,1]$, it is the asymptotic distribution of numbers in the nth Farey sequence as $n \to \infty$ and it can be given as the fixed point of an iterated function system consisting of two rational functions.

Our interest is in the (monic) orthogonal polynomials $(P_n)_{n\in\mathbb{N}}$ for the Minkowski measure q and in particular in the behavior of the recurrence coefficients in their three term recurrence relation

$$xP_n(x) = P_{n+1}(x) + b_n P_n(x) + a_n^2 P_{n-1}(x).$$

The symmetry of the question mark function gives $b_n = 1/2$ for all $n \geq 0$. The behavior of a_n^2 is more complicated and is still an open problem. We will show some numerical experiments using the Stieltjes-Gautschi method with a discrete measure supported on the Farey sequence. We also explain how one can compute the moments of the measure q, from which one can also compute the recurrence coefficients. This is however a badly conditioned problem and does note allow the computation of sufficiently many a_n^2 to draw some conclusions about their asymptotic behavior.

Short Talks

January 20

	Plenary room	
9:30-10:30	A. Bultheel	
10:30-11:30	W. Van Assche	
11:30-12:00	Break	
	Room 1	Room 2
12:00-12:30	Li. Garza	M. Castro
12:30-13:00	Lassarov	A. Martínez
13:00-13:30	X. Li	V. Sánchez
13:30-15:30	Lunch	
	Plenary room	
15:30-16:30	A. Aptekarev	
16:30-17:00	Break	
	Room 1	Room 2
17:00-17:30	J.S. Dehesa	Rebocho
17:30-18:00	R. Alvarez	Neuschel
18:00-18:30	S. Medina	Pejcev

On Computational Aspects of Discrete Sobolev Inner Products on the Unit Circle.

Lino Gustavo Garza Gaona
Universidad Carlos III de Madrid

Abstract

In this contribution, we show how to compute in $O(n^2)$ steps the Fourier coefficients associated with the Gelfand-Levitan approach for discrete Sobolev orthogonal polynomials on the unit circle when the support of the discrete component involving derivatives is located outside the closed unit disk. As a consequence, we deduce the outer relative asymptotics of these polynomials in terms of those associated with the original orthogonality measure. Moreover, we show how to recover the discrete part of our Sobolev inner product.

On a seminal paper by Karlin and McGregor

Mirta M. Castro Smirnova
Universidad de Sevilla

Abstract

The seminal paper by S. Karlin and J. McGregor [1] connects orthogonal polynomials and birth-and-death processes. Explicit results for the orthogonality measure and the orthogonal polynomials are given in two cases resulting from special relations among the parameters controlling the boundary condition at the origin. Here we allow for a general set of parameters and give the corresponding measure and orthogonal polynomials.

References:

[1] S. Karlin and J. McGregor "Random walks", Ilinois J. Math., **3** (1959), pp. 66–81.

[2] M. Castro and F.A. Grünbaum, "On a seminal paper by Karlin and McGregor", Symmetry Integrability and Geometry: Methods and Applications (SIGMA), 9 (2013), 020, 11 pages.

The Eneström–Kakeya theorem encounters the theory of orthogonal polynomials on the unit circle

Andreas Lasarow
HTWK Leipzig

Abstract

A classical result due to Eneström and Kakeya gives some bounds for the moduli of the zeros of polynomials having a monotone sequence of non-negative (real) coefficients. The main subject of the talk is a study of this fact with a view to the recurrence relations fulfilled by systems of orthogonal polynomials on the unit circle. In particular, we will be interested in the special case, where the zeros of the polynomials in question are not located on the boundary of the estimate which occurs in the Eneström–Kakeya theorem. Among other things, we will give characterizations of this case in terms of orthogonal polynomials. Furthermore, we will give some insight how one can apply the main results of this paper in the context of positive Hermitian Toeplitz matrices.

Matrix-Valued Gegenbauer Polynomials

Ana Martínez de los Ríos
Universidad de Sevilla

Abstract

For $\nu \in (0, \infty)$ we construct a family of weight matrices $\left(W^{(\nu)}(x)\right)_\nu = (1 - x^2)^{\nu - 1/2} M^{(\nu)}(x)$, with common support $[-1, 1]$. These weights can be seen as matrix analogues to the scalar Gegenbauer polynomials. We will prove that for $\nu > 0$ one can always find two matrix polynomials $\Phi^{(\nu)}(x)$, $\Psi^{(\nu)}(x)$ such that the following is satisfied,

$$W^{(\nu+1)}(x) = W^{(\nu)}(x)\Phi^{(\nu)}(x), \quad W^{(\nu)}(x)|_{x=-1,1} = 0, \quad \nu > 0,$$
$$\left(W^{(\nu)}(x)\Phi^{(\nu)}(x)\right)' = W^{(\nu)}(x)\Psi^{(\nu)}(x), \quad x \in [-1, 1].$$

What assures that the families of monic orthogonal polynomials with respect to $W^{(\nu)}$ and $W^{(\nu+1)}$ are related by

$$\left(\frac{d}{dx} P_n^\nu(x)\right)_{n \geq 1} = \left(n P_{n-1}^{\nu+1}\right)_{n \geq 0}.$$

Some formulas and properties for the weight matrices and for the matrix polynomials will also be discussed.
This is a Joint work of Erik Koelink (Radboud University), Pablo Román (Universidad Nacional de Córdoba) and Ana M. de los Ríos (Universidad de Sevilla).

On Finite Blaschke Products Interpolating on the Unit Circle

Xin Li
University of Central Florida

Abstract

Given n distinct points z_j, $j = 1, 2, ..., n$, on the unit circle in the complex plane and given n values w_j, $j = 1, 2, ..., n$, also on the unit circle, it is known that there exist finite Blaschke products B_n with at most n poles

$$B_n(z) = \prod_{j=1}^{n} \frac{1 - \bar{a}_j z}{z - a_j}$$

such that

$$B_n(z_j) = w_j, \ \ j = 1, 2, ..., n.$$

We will discuss the existence and uniqueness questions for this interpolation problem. In the case of non-uniqueness, due to its nonlinear nature, the poles of all possible solutions are hard to characterize. C. Glader used Nevanlinna parametrization to describe all solutions. We will show a simpler set of parameters that may be employed to give a complete parametrization of the set of all the solutions. This talk will be based on joint research with R.N. Mohapatra and R. Puwakgolle.

Rodrigues formulas for orthogonal matrix polynomials satisfying second order difference equations

Vanesa Sánchez Canales
Universidad de Sevilla

Abstract

In this talk, we show a method to find Rodrigues formulas for orthogonal matrix polynomials satisfying second order difference equations. We also show some illustrative examples.

Joint work with Antonio J. Durán. Universidad de Sevilla

Hankel-transform inequalities, hyperspherical harmonics and entropic uncertainty relations

Jesus S. Dehesa
Universidad de Granada

Abstract

Recent inequalities of the Hankel transform of L. De Carli together with various properties of the hyperspherical harmonics are used to improve some entropic uncertainty relations for D-dimensional spherically-symmetric potentials. These relations are mathematical formulations of the quantum Uncertainty Principle which generalize the celebrated variance-based Heisenberg relation. Comparison with the elegant general relations of Rajagopal and Maassen-Uffink is done. Finally, some specific applications are shown.

Deformed Laguerre-Hahn orthogonal polynomials on the real line

Maria das Neves Rebocho
University of Beira Interior

Abstract

This talk focuses on sequences of orthogonal polynomials on the real line related to Stieltjes functions that satisfy a Riccati-type differential equation with polynomial coefficients. We deduce discrete Lax equations which lead to difference equations for the corresponding three term recurrence relation coefficients, and we analyze the continuous differential equations that arise when deformations under a t-dependence on the coefficients of the Riccati differential equation occur. This is joint work with A. Branquinho (CMUC and Department of Mathematics, University of Coimbra, Portugal).

On linearly related sequences of difference derivatives of discrete orthogonal polynomials and some applications

Renato Álvarez-Nodarse
Universidad de Sevilla

Abstract

Let ν be either $\omega \in \mathbb{C} \setminus \{0\}$ or $q \in \mathbb{C} \setminus \{0, 1\}$, and let D_ν be the difference operator defined either by $D_\omega p(x) = \frac{p(x+\omega)-p(x)}{\omega}$ or $D_q p(x) = \frac{p(qx)-p(x)}{(q-1)x}$.

Let $\mathbb{U}$ and $\mathbb{V}$ be two moment regular functionals and let $(P_n)_n$ and $(Q_n)_n$ be their corresponding orthogonal polynomial sequences (OPS) that satisfy the following linear algebraic structure relation

$$\sum_{i=0}^{M} a_{i,n} D_\nu^m P_{n+m-i}(x) = \sum_{i=0}^{N} b_{i,n} D_\nu^k Q_{n+k-i}(x), \quad n \geq 0,$$

where $M, N, m, k \in \mathbb{N} \cup \{0\}$, $a_{M,n} \neq 0$ for $n \geq M$, $b_{N,n} \neq 0$ for $n \geq N$, and $a_{i,n} = b_{i,n} = 0$ for $i > n$.

In this talk we will show that, under some natural conditions, $\mathbb{U}$ and $\mathbb{V}$ are related by a rational factor (in the $\nu-$distributional sense). Moreover, when $m \neq k$ then both $\mathbb{U}$ and $\mathbb{V}$ are D_ν-semiclassical functionals. Finally, an application to the OPS with respect to the following Sobolev-type inner product

$$\langle p(x), r(x) \rangle_{\lambda,\nu} = \langle \mathbb{U}, p(x)r(x) \rangle + \lambda \langle \mathbb{V}, (D_\nu^m p)(x)(D_\nu^m r)(x) \rangle, \quad \lambda > 0,$$

will be shown. This is a joint work with J. Petronilho, N. C. Pinzón-Cortés and R. Sevinik-Adıgüzel.

On a conjecture on sparse binomial-type polynomials

Thorsten Neuschel
KU Leuven

Abstract

We prove a conjecture by Brown, Dilcher and Manna on the asymptotic behavior of sparse binomial-type polynomials arising naturally in a graph theoretical context in connection with the expected number of independent sets of a graph.

On the convergence of Hermite-Padé approximants

Sergio Medina Peralta
Universidad Carlos III de Madrid

Abstract

The convergence of sequences of type I and type II Hermite-Padé approximants for a certain class of vector functions is considered. This class of functions is obtained perturbing a Nikishin system with polynomials or rational functions. We obtain extensions of Markov's and Stieltjes' theorem.

Error bounds of Micchelli-Rivlin quadrature formula for analytic functions

Aleksandar Pejcev
Faculty of Mechanical Engineering, Serbia

Abstract

We consider the well known Micchelli-Rivlin quadrature formula, of highest algebraic degree of precision, for the Fourier-Chebyshev coefficients. For analytic functions the remainder term of this quadrature formula can be represented as a contour integral with a complex kernel. We study the kernel, on elliptic contours with foci at the points ∓ 1 and a sum of semi-axes $\rho > 1$, for the quoted quadrature formula. Starting from the explicit expression of the kernel, we determine the locations on the ellipses where maximum modulus of the kernel is attained. So we derive effective L^∞-error bounds for this quadrature formula. Complex-variable methods are used to obtain expansions of the error in the Micchelli-Rivlin quadrature formula over the interval $[-1, 1]$. Finally, effective $L1$-error bounds are also derived for this quadrature formula.

January 21

	Plenary room	
9:30-10:30	A. Martínez Finkelshtein	
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11:30-12:00	Break	
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16:30-17:00	Break	
	Room 1	Room 2
17:00-17:30	Derevyagin	C. Pestano
17:30-18:00	D. Rivero	Berriochoa
18:00-18:30	Štampach	J. Illán

Inverse resonance problem for perturbations of periodic Jacobi matrices: existence, uniqueness, stability

Rostyslav Kozhan
KTH, Stockholm

Abstract

We characterize spectral measures of exponentially decaying and finite range perturbations of periodic Jacobi matrices. This result is new even for the perturbations of the free Jacobi matrix. As a corollary of this characterization, we are able to fully solve the inverse resonance problem. Namely, we determine when a system of points is the sequence of resonances of a Jacobi matrix, when such a Jacobi matrix is unique, and we establish stability properties of this inverse problem.

On asymptotic properties of multivariate orthogonal polynomials

Luis E. Garza
Universidad de Colima

Abstract

We consider polynomials in several variables orthogonal with respect to a Sobolev-type inner product, obtained from adding a higher order gradient evaluated in a fixed point to a standard inner product. An expression for these polynomials in terms of the orthogonal family associated with the standard inner product is obtained. A particular case using polynomials in the unit ball is analyzed, and some asymptotic results are derived. Joint work with Herbert Dueñas (Universidad Nacional de Colombia) and Miguel Piñar (Universidad de Granada)

On some extremal problems of approximation theory of ridge functions

Vugar Ismailov
Institute Azerbaijan National Academy of Sciences

Abstract

Ridge functions are multivariate functions of the form

$$g\left(a_1x_1+\ldots+a_dx_d\right)=g\left(\mathbf{a}\cdot\mathbf{x}\right),$$

where $g:\mathbb{R}\to\mathbb{R}$ and $\mathbf{a}=(a_1,...,a_d)$ is a fixed vector (direction) in $\mathbb{R}^d\backslash\{\mathbf{0}\}$. In other words, ridge functions are multivariate functions constant on the parallel hyperplanes $\mathbf{a}\cdot\mathbf{x}=\alpha,\ \alpha\in\mathbb{R}$. These functions and their linear combinations arise naturally in problems of computerized tomography, statistics, partial differential equations (where they are called *plane waves*), neural networks, and approximation theory. We consider the problem of uniform and L_2 approximation of a continuous multivariate function $f(\mathbf{x})=f(x_1,...,x_d)$ by linear combinations of ridge functions. We give a necessary and sufficient condition for a sum of ridge functions to be a best approximation (extremal element) to $f(\mathbf{x})$. The main results are next used to construct a best approximation and to obtain an explicit formula for the approximation error. The problem of well approximation by such sums is also considered.

Asymptotic expansion for a class of ${}_2F_1$–functions.

Jorge Alberto Borrego Morell
Universidad Estadual Paulista

Abstract

For the class of bi–orthogonal hypergeometric polynomials on the unit circle with respect to the complex valued weight $(1-e^{i\theta})^{\alpha+\beta}(1-e^{-i\theta})^{\alpha-\beta}$ we give an non uniform asymptotic expansion in compact subsets of the complex plane including error bounds and some other properties as well. A class of Szegö polynomial is also considered.

Approximation by Delsarte translates of a basis function

Cristian Arteaga
Universidad de La Laguna

Abstract

Given a basis function K, let $\mathcal{S}_1(K)$ consist of all those functions $v : \mathbb{R}_+ \to \mathbb{R}$ which can be represented as

$$v(x) = \sum_{i=1}^{m} w_i \tau_{z_i}(\lambda_{\sigma_i} K)(x) \quad (x \in \mathbb{R}_+), \tag{1}$$

where $m \in \mathbb{N}$, $w_i \in \mathbb{R}$, $z_i \in \mathbb{R}_+$ and $\sigma_i > 0$ ($i \in \mathbb{N}$, $1 \leq i \leq m$). Here, $\tau_z = \tau_{\mu,z}$ ($z \in \mathbb{R}_+$) stands for the Delsarte translation operator of order $\mu > -1/2$, and $(\lambda_\sigma K)(t) = K(t/\sigma)$ ($t \in \mathbb{R}_+,\ \sigma > 0$) is a dilation operator. Let $\mathcal{S}_0(K)$ denote the subfamily of $\mathcal{S}_1(K)$ for which $\sigma_i = \sigma$ ($\sigma > 0,\ i \in \mathbb{N},\ 1 \leq i \leq m$), and let $\mathcal{S}(K)$ denote the subfamily of $\mathcal{S}_0(K)$ for which $\sigma = 1$. Using a variety of techniques, both constructive and nonconstructive, we give necessary and/or sufficient conditions on the kernel K ensuring that $\mathcal{S}(K)$, $\mathcal{S}_0(K)$ or $\mathcal{S}_1(K)$ are dense in spaces of p-integrable functions, $1 \leq p < \infty$, or in spaces of continuous functions. In the case of $\mathcal{S}(K)$, this approach yields analogues of the celebrated Wiener and Wiener-Pitt tauberian theorems, with the Fourier-Bessel transformation replacing the Fourier one.

This is a joint work with Isabel Marrero.

Asymptotic expansions for generalised Jacobi polynomials

Peter Opsomer
KU Leuven

Abstract

We (P. Opsomer, A. Deaño and D. Huybrechs from KU Leuven) are interested in the asymptotic behavior of orthogonal polynomials of the generalised Jacobi type as their degree n goes to ∞. These are defined on the interval $[-1, 1]$ with weight function

$$w(x) = (1-x)^{\alpha}(1+x)^{\beta}h(x), \quad \alpha, \beta > -1$$

and $h(x)$ a real, analytic and strictly positive function in a sizeable neighbourhood of $[-1, 1]$. This information is available in the work of Kuijlaars, McLaughlin, Van Assche and Vanlessen, where the authors use the Riemann–Hilbert formulation and the steepest descent method. An efficient way to compute higher order terms and asymptotic expansions in every region of the complex plane are derived. This is implemented symbolically in Maple and numerically in Matlab and will be made publicly available.

Definitizability, Darboux transformations, and Stahl's counterexample

Maxim Derevyagin
KU Leuven

Abstract

We will discuss relations between spurious poles at infinity for Cauchy transforms of signed measures and definitizability, the concept, which in some sense extends self-adjointness for operators in Krein spaces. It will be also shown that Darboux transformations naturally lead to definitizable operators. All these facts will be illustrated by Stahl's counterexample.

A Note on Algebraic Properties of Matrix Padé Approximants

Pestano-Gabino
Universidad de La Laguna

Abstract

Although Padé approximation is born with the idea of improving the approximation of functions, in this work is not used to that. The algebraic properties have allowed us, among other things, to detect certain errors in papers of another field, that of the VARMA models. Stripping those comments of the context in which it is written, translating it into the context of matrix rational functions and using certain algebraic properties of the approximants lead us to find suitable examples to illustrate our claims.

This is a joint work with C. González-Concepción, M.C. Gil-Fariña

Edge Detection Based on Krawtchouk Polynomials

Daniel Rivero
Universidad Politécnica de Madrid

Abstract

Discrete orthogonal polynomials are useful tools in digital image processing to extract visual object contours in different application contexts. This paper proposes a method that extends beyond classic first-order differential operators, by using the properties of Krawtchouk orthogonal polynomials and analysis of zero crossings to achieve a second order differential operator with higher order precision. Experimentally, we provide simulation results which prove that the proposed method achieves superior performance in comparison with commonly used algorithms.

Interpolation of order two on the unit circle: Convergence.

Elias Berriochoa Esnaola
Universidad de Vigo

Abstract

We deal with Hermite interpolation problems on the unit circle considering up to the second derivative for the interpolation conditions and taking equally spaced points as nodal system. In the extended Fejér case, which corresponds to take vanishing values for the first two derivatives, we prove the uniform convergence for the interpolants related to continuous functions with smooth modulus of continuity. We also consider the Hermite case with non vanishing conditions for the derivatives for which we establish sufficient conditions on the interpolation conditions to obtain convergence.

This is a joint work with Alicia Cachafeiro and Jaime Díaz.

Nevanlinna functions and orthognality relations for q-Lommel polynomials

František Štampach
Department of Applied Mathematics, Faculty of Information Technology,
Prague, Czech Republic.

Abstract

Some q-Lommel polynomials arise naturally from the three term recurrence for Hahn-Exton q-Bessel functions. They have been intensively studied by Koelink and others in 90's. The corresponding Hamburger as well as the Stieltjes moment problem is both determinate and indeterminate according to some restrictions on involved parameters. Koelink described the measure of orthogonality in the determinate case which is at the same time one N-extremal measure in the indeterminate case. The measure can be described in terms of Hahn-Exton q-Bessel functions.

In this talk, we provide formulas for all four functions from the Nevanlinna parametrization in the case of indeterminate Hamburger moment problem. By using these formulas we can describe all N-extremal measures and also some absolutely continuous measures of orthogonality for q-Lommel polynomials. If there is some time left we discuss some recurrence relations for the corresponding moment sequence.

Convergence, numerical implementation and some applications of Gauss quadrature formulae associated with nearly singular modifications

J. Illán
Universidad de Vigo

Abstract

Let $\int_{-1}^{1} F(x)W(x)dx$, where F is nearly singular, $W(x) = p(x)/\sqrt{1-x^2}$, and p is a polynomial. Suppose that $F = fG/q$, where f varies smoothly, q is a polynomial whose zeros coincide with difficult poles of F, and G is nearly singular but this feature is not related to the poles of F. The task we have to face is to integrate f w.r.t. GW/q using the corresponding Gauss formula. In one of his books, W. Gautschi suggested that this way of separating the singularities of F could be convenient, but therein we found neither about the calculation of the quadrature nor about convergence. This talk attempts to summarize a research that has been conducted by the speaker and several colleagues, in connection with the approximate calculation of nearly and strongly singular integrals. It is shown how Gautschi's ideas can be redirected in order to obtain convergence and facilitate the calculation of the quadrature parameters. The technique that we follow to establish some convergence results is simple and consists in to replace G by a suitable sequence of polynomials $\{G_n\}$, with $G_n \to G$ in L^1-norm. For numerical purposes, we consider only the factor G, by assuming that it contains all the singularities of F, including poles, if any. To carry out this focus we use the modified moments algorithm and its connection with the Chebyshev series of G to obtain exact formulas for nodes and weights. The coefficients of this series are approximated using FFT or Chebyshev interpolation formula. The results obtained using this method are compared with those produced by other quadrature formulas whose high efficiency is unquestionable. As an application we show how this approach can also be used to calculate Hadamard finite-part integrals.
This research was carried out jointly with A. Cachafeiro, E. Berriochoa. In addition, F. Cala and J. M. Rebollido have also been involved in different stages of this work.

January 22

	Plenary room
9:30-10:30	M. J. Cantero
10:30-11:30	B. de la Calle
11:30-12:00	Break
12:00-18:30	Excursion to las Cañadas (including a visit to Observatory)

Figure 2: Mt. Teide Astrophysics Observatory

January 23

	Plenary room
9:30-10:30	R. Cruz Barroso
10:30-11:30	F. Perdomo Pío
11:30-12:00	Break
12:00-12:30	A. Kuijlaars
12:30-13:00	E. Medina
13:00-13:30	J. Sánchez
13:30-15:30	Lunch
15:30-16:00	G. Silva
16:00-16:30	A. Deaño
16:30-17:00	Break
17:00-17:30	V. Kalyagin
17:30-18:00	Y. Xu
18:00-18:30	M. Piñar

	Special Session
12:00-12:20	José C. Sabina de Lis
12:20-12:40	Antonio Bonilla Ramírez
12:40-13:00	Teresa Bermúdez de León
13:00-13:20	Matías Camacho Machín
13:20-13:40	Luis Moreno-Armella
13:40-15:30	Lunch
15:30-15:50	Juan Carlos Fariña Gil
15:50-16:10	Domingo Hernández Abreu
16:10-16:30	Severiano González Pinto
16:30-16:50	Manuel Linares Linares
16:50-17:30	Break
17:30-18:00	Rafael Montenegro Armas
18:00-18:30	Manuel Calvo Pinilla

Multiple orthogonal polynomials and products of random matrices

Arno Kuijlaars
KU Leuven

Abstract

The talk is based on joint work with Lun Zhang [2] Recently, Akemann et al. [1] showed that squared singular values of products of random matrices with independent complex Gaussian entries give rise to a determinantal point process whose correlation kernel is given in terms of Meijer G-functions. The determinantal point process is in fact a multiple orthogonal polynomial ensemble. We describe some of the properties of this new class of multiple orthogonal polynomials. For the case of a product of two matrices they lead to multiple orthogonal polynomials with modified Bessel weights that were first studied by Van Assche and Yakubovich in [3].

References:

[1] G. Akemann, J.R. Ipsen and M. Kieburg, Products of rectangular random matrices: singular values and progressive scattering, Physical Reveiw E 88, 052118 (Nov. 2013).

[2] A.B.J. Kuijlaars and L. Zhang, Singular values of products of Ginibre random matrices, multiple orthogonal polynomials and hard edge scaling limits, preprint, arXiv: 1308.1003

[3] W. Van Assche and S.B. Yakubovich, Multiple orthogonal polynomials associated with Macdonald functions, Integral Transforms Spec. Funct. 9 (2000), 229–244

Determination of S-curves and phase structure of zero densities of orthogonal polynomials

Elena Medina
Universidad de Cádiz

Abstract

The goal of the talk is the determination of S-curves in the theory of non-Hermitian orthogonal polynomials in the complex plane with respect to exponential weights. The S-property leads us to the spectral curve, and thus, the fact that the branch points of the cuts (support of the asymptotic zero density) can be written in terms of periods of Abelian differentials on a suitable Riemann surface, allows us to analyze the phase structure of asymptotic zero densities of orthogonal polynomials. As an application, we give a complete description of the phases and critical processes of the standard cubic model.

On the dynamics of the equilibrium measure for some rational external fields

Joaquín F. Sánchez-Lara
Universidad de Granada

Abstract

The subject of the present talk is the study of families of equilibrium measures in the real line in the presence of rational external fields. It is well known that the support of an equilibrium measure in a real analytic external field is comprised of a finite number of intervals. In the last years, many papers have dealt with equilibrium problems in the presence of polynomial external fields, paying special attention to the evolution of the support of the equilibrium measure when the total mass of the measure (also regarded as the "time" or "temperature") varies in $(0, +\infty)$. In the present talk, we extend this study to the case of certain rational fields (that is, when the derivative of the field, φ' is a rational function), where a polynomial field is perturbed with the addition of a logarithmic term of the form $\log(x^2 + \nu)$, $\nu > 0$. This kind of equilibrium problems is present on a number of physical problems related to random matrix models. The situation when $\nu \to 0+$, which leads to the so-called generalized Gaussian-Pener models, it is also considered. This is a joint work with Z. García and R. Orive (Universidad de La Laguna)

S-property in polynomial external field

Guilherme Silva
KU Leuven

Abstract

Guilherme Silva (KU Leuven) Consider a sequence of polynomials (P_n) satisfying the (non-hermitian) complex orthogonality

$$\int_\Gamma z^j P_n(z) e^{-nV(z)} dz = 0, \ j = 0, \dots, n-1,$$

where V is a fixed polynomial and the integration is on an unbounded simple contour Γ in $\mathbb{C}$ ending up at ∞ in both directions and such that $\Re V(z) \to +\infty$, as $z \to \infty$ in Γ. Gonchar and Rakhmanov [1] characterized the limiting distribution of the zeroes of these polynomials, conditioned to the existence of a curve Γ with a certain symmetry property - the so called *S-property* - over which we can compute the integrals above. We discuss the existence of this curve Γ and its characterization. Our approach is strongly based on recent works of Rakhmanov [3] and Martínez-Finkelshtein and Rakhmanov [2]. Following the approach suggested on [3], we consider a max-min energy problem on a suitable class of contours. We prove that this problem has a solution and this solutions possesses the S-property. This is a joint work with Arno Kuijlaars (KU Leuven).

References:

[1] A. A. Gonchar and E. A. Rakhmanov, *Equilibrium distributions and the rate of rational approximation of analytic functions*, Mat. Sb. (N.S.) 134(176) (1987), no. 3, 306–352, 447.

[2] A. Martínez-Finkelshtein and E. A. Rakhmanov, *Critical measures, quadratic differentials, and weak limits of zeros of Stieltjes polynomials*, Comm. Math. Phys. 302 (2011), no. 1, 53–111.

[3] E. A. Rakhmanov, *Orthogonal polynomials and S-curves*, Contemp. Math., vol. 578, Amer. Math. Soc., Providence, RI, 2012.

Asymptotic analysis of polynomials orthogonal with respect to an oscillatory weight function

Alfredo Deaño Cabrera
KU Leuven/ Universidad Carlos III de Madrid

Abstract

We are interested in the asymptotic behavior of polynomials $P_n(x)$ that are formally orthogonal with respect to the oscillatory weight function $w(x) = e^{i\omega x}$ on $[-1, 1]$. The behavior of $P_n(x)$ and in particular the distribution of the roots as $\omega \to \infty$ was analyzed in the work of Asheim, Deaño, Huybrechs and Wang (Disc. Cont. Dyn. Syst. 34, 3 (2014), 883-901), motivated by the construction of complex quadrature rules with Gaussian properties. The behavior of $P_n(x)$ as $n \to \infty$ can be analyzed using the Riemann-Hilbert formulation and the results of Kuijlaars, McLaughlin, Van Assche and Vanlessen on Jacobi-type weights (Adv. Math. 188 (2004), 337-398). The behavior as the two parameters n and ω are simultaneously large and coupled in different regimes will be discussed.

On a numerical stability of modified Chebychev algorithm for multiple orthogonal polynomials and generalized Volterra lattice

Valery Kalyagin
National Research University Higher School of Economics, Nizhny Novgorod Maksim Sokolov

Abstract

Volterra or Langmuir lattice is one of dynamical models where the interaction of particle with the nearest neighbors is taken into account. It is known since J.Moser that the analysis of the Volterra lattice is related with isospectral deformation of a tridiagonal Jacobi operator. The main numerical problem in this setting is the inverse spectral problem for the Jacobi operator. This problem is related with system of polynomials orthogonal with respect to the spectral measure of operator. Generalized Volterra lattice is a dynamical model where the interaction of particle with some fixed number of neighbors is taken into account. This model is a particular case of discrete KP equation. The analysis of discrete KP equation is related with Hessenberg operator, multiple orthogonal polynomials and Hermite-Padé approximants of the resolvent functions of operator. To solve the inverse spectral problem for Hessenberg operator a modified Chebychev algorithm can be applied. In the paper we discuss the numerical stability of modified Chebychev algorithm for multiple orthogonal polynomials. As a result we find the class of perturbation of spectral measure where the algorithm is stable. Some applications to generalized Volterra lattice are given.

Approximation and orthogonal polynomials in Sobolev spaces

Yuan Xu
University of Oregon

Abstract

For the spectral Galerkin method in numerical solution of partial differential equations, we need to understand approximation by polynomials in the Sobolev spaces. For this purpose, it is necessary to study orthogonal structure of the Sobolev space W_2^r that consists of functions whose derivatives up to r-th order are all in L^2. In this talk, we discuss new result on Sobolev orthogonal polynomials in W_2^r for all positive integer r on the unit ball in R^d, describe sharp estimate for the error of best approximation in the Sobolev space and its application in the spectral Galerkin methods.

Two–variable analogues of Jacobi polynomials

Miguel A. Piñar
Universidad de Granada

Abstract

We study two–variable Jacobi polynomials on different domains. Using the Koornwinder's product formula for Jacobi polynomials we deduce old and new representation formulae for the corresponding kernels. As a consequence, asymptotic results for the Christoffel functions are obtained.

Special Session. Mathematics: Theory and Applications.

Diffusion problems where multiple solutions arise

José C. Sabina de Lis
Universidad de La Laguna

Abstract

Some results describing the existence of multiple positive solutions to a class of nonlinear diffusion problems will be presented. Furthermore, existence of two-signed solutions will be also discussed

Chaos in Linear Dynamics

Antonio Bonilla
Universidad de La Laguna

Abstract

We study the Li-Yorke chaos, the distributional chaos and the Devaney chaos for linear operators on Banach and Frechet spaces. Sufficient "computable" criteria for Li-Yorke, distributional and Devaney chaos are given.

References:

[1] T. Bermúdez, A. Bonilla, F. Martínez-Giménez and A. Peris, Li-Yorke and distributionally chaotic operators, J. Math. Anal. Appl., 373, (2011), 83-93.

[2] N. C. Bernardes, A. Bonilla, V. Muller and A. Peris, Distributional chaos for linear operators, J. Funct. Anal.,265,(2013), 2143-2163.

[3] N. C. Bernardes, A. Bonilla, V. Muller and A. Peris, Li-Yorke chaos in linear dynamics, Preprint.

On m-isometries.

Teresa Bermúdez
Universidad de La Laguna

Abstract

In this talk we give sufficient conditions to be an ℓ-isometry, that is, given an m-isometry A and a nilpotent operator Q of order n that commutes, then $A+Q$ is a $(2n+m-1)$-isometry. As an application we give examples of m-isometries and prove that $A+Q$ cannot be N-supercyclic for any N, where A is an isometry. Also, we find an m-isometry with prescribed spectrum K, where K is the closed unit disk or a compact subset of the unit circle.

This a Joint Work with Antonio Martinón, Vladimir Müller and Juan Agustín Noda

A Dynamic Approach to Problem Solving Activities in the preparation of Secondary School Mathematics Teachers

Matías Camacho Negrín
Universidad de La Laguna

Abstract

Mathematical tasks are essential components that help prospective and practicing teachers to develop mathematics and didactic knowledge. What type of reasoning could problem solvers exhibit when they use a dynamic software to solve textbook tasks? In this study, we focus on analyzing the problem solving sessions developed within a community formed by mathematicians, mathematics educators and prospective and in-service high school teachers which aimed to characterize and discuss ways of thinking and reasoning that the members of the community showed while using a dynamic software (Geogebra) to represent, explore, and solve a set of textbook problems. We analyse and discuss the extent to which the systematic use of the tool offers prospective teachers the opportunity to construct and explore dynamic models of textbook problems in terms of visual, empirical, and geometric reasoning. In this context, we report that the use of the tool not only offers prospective teachers novel ways to think of the tasks, but also the nature of routine problems can be transformed into a series of non-routine problem solving activities.

This is a joint work with Manuel Santos-Trigo (Cinvestav IPN, México) and Mar Moreno-Moreno (University of Lleida, Spain).

Intuition and Rigor: A never-ending dance

Luis Moreno-Armella
Cinvestav-IPN, México

Abstract

There is a problem that goes through the history of calculus: The tension between the intuitive and the rigorous. Calculus continues to be taught as if it were natural to introduce the study of change and accumulation by means of formalized ideas and concepts known as the mathematics of ϵ and δ. It is frequently considered as a failure that "students still seem to conceptualize limits via the imagination of motion." This kind of assertions shows the tension, the rift created by traditional education between students' intuitions and a misdirected formalization. In fact, the internal connections of the intuition of change and accumulation are not correctly translated into that arithmetical approach of ϵ and δ. There are other routes to formalization and perhaps another formalization, which coheres with these intuitions. I intend to discuss these issues in my talk. My departing point is epistemic and once this discussion is put forward, I intend to produce a glimpse into the didactic field.

Weak type (1,1) estimates for Caffarelli-Calderón generalized maximal operators for semigroups associated with Bessel and Laguerre operators

Juan Carlos Fariña
Universidad de La Laguna

Abstract

In this talk we prove that the generalized (in the sense of Caffarelli and Calderón) maximal operators associated with heat semigroups for Bessel and Laguerre operators are weak type $(1,1)$. Our results include other known ones and our proofs are simpler than the ones for the known special cases.

This is a joint work with Jorge Betancor, Alejandro Castro, Pablo de Napoli and Lourdes Rodríguez-Mesa

On a family of collocation Runge-Kutta Methods based on quadrature rules of order $2s-3$ with two prescribed nodes

Domingo Hernández-Abreu
Universidad de La Laguna

Abstract

The stability and convergence properties of a recently introduced one-parameter family of high order strongly A-stable Runge-Kutta collocation methods with a first internal stage of explicit type are discussed. The so-called $SAFERK(\alpha, s)$ methods, with free parameter α and s internal stages, are based on interpolatory quadrature rules with precision degree equal to $2s-4$, and all of them have two prefixed nodes, $c_1 = 0$ and $c_s = 1$. The methods are well-suited for the numerical integration of stiff and differential-algebraic systems, and their implementation involve the same amount of implicitness as for the $(s-1)$-stage *Radau IIA* method. For the same number of implicit stages, both $SAFERK(\alpha, s)$ and *Radau IIA*$(s-1)$ methods possess algebraic order $2s-3$, whereas the stage order is one unit higher for *SAFERK* methods. Furthermore, the free parameter α can be selected in order to minimize the principal term of the local error or to maximize the numerical dissipation. On the other hand, it is shown how the 4-stage methods can be endowed with an embedded third order formula, and an implementation with an adaptive stepsize controller proves to be competitive for a wide selection of test problems coming from electric circuit analysis, constrained mechanical systems, and time-dependent partial differential equations treated by the method of lines.

This is a joint work with S. González-Pinto and J.I. Montijano

Rosenbrock-AMF methods for time dependant Advection Diffusion Reaction equations

Severiano González-Pinto
Universidad de La Laguna

Abstract

Rosenbrock-type methods for the numerical integration of the ODE systems resulting of spatial semidiscretization of PDEs of Advection Diffusion Reaction in the MoL framework are considered. The spatial discretization is based on Finite Differences and the time integration is carried out by using AMF-splitting (Approximate Matrix Factorization) applied to some Rosenbrock-type method. Some refinements to the usual Approximate Matrix Factorization (AMF) are studied. These AMF-refinements allow to recover the convergence order of the underlying method and in some cases to enlarge the linear stability regions and the Courant numbers with regard to the standard AMF-scheme. Most of these methods belong to the class of the W-methods (named after Steihaug and Wolfbrandt, 1979). A few numerical experiments on some important 2D and 3D non-linear PDE problems with applications in Physics are presented.

This is a joint work with D. Hernández-Abreu and S. Pérez-Rodríguez

Mathematics in Finance: Concepts, Techniques, Tools and Applications

Manuel Linares Linares
Universidad de La Laguna

Abstract

Over the last 40 years,"DERIVATIVES" have become increasingly important in Finance. My key objective is to help you to understand this new concept, whether you love "DERIVATIVE" or hate them, you can-not ignore them at least for the next few years.

We shall speak about different types of "Derivatives": Options (Call, Put, American, Asian,...), Futures, Forward, FRA, Bonds, Swaps, and others. There are very important issues connected with them and in particular, Pricing, Hedging, Portfolios, Derivatives of Interest Rates, Risk, Quantitative Risk Management, CDS and Modeling stand out. As you can imagine, in order to gain vital knowledge of these concepts, a lots of mathematics will be needed, Probability, Conditional Expectations, Martingales, Stochastic Processes, S.D.E. and Statistics

Our main goal is to derive closed-form expressions for both the price of the options and the replicating strategy in the Black-Scholes setting. This permits us to obtain the well- known Black-Scholes Option Valuation Formula.

There have been several crises recently. In the wake of these, the topic of Quantitative Risk Management is high on the agenda of academics, practitioners, regulators, politicians, the media, as well as the public at large. This new theory permits us to constructs models which go beyond normal dependence, where the concept of Copula is fundamental.

We finish with some remarks about The Most Important Derivatives Markets

The Meccano method for isogeometric analysis of planar domains

Rafael Montenegro
Universidad de Las Palmas de Gran Canaria

Abstract

The authors have recently introduced the meccano method for tetrahedral mesh generation and volume parameterization of solids. In this paper, we present advances of the method for T-spline modelling and analysis of complex geometries. We consider a planar domain composed by several irregular sub-domains. These sub-regions are defined by their boundaries and can represent different materials. The bivariate T-spline representation of the whole physical domain is constructed from a square. In this procedure, a T-mesh optimization method is crucial. We show results of an elliptic problem by using a quadtree local T-mesh refinement technique.

This is a joint work with M. Brovka, J.I. López, J.M. Escobar, J.M. Cascón and G. Montero.

The effect of exponential fitting on the stability of numerical methods

Manuel Calvo Pinilla
Universidad de Zaragoza

Abstract

In the numerical solution of Initial Value Problems (IVPs) for differential systems, exponential fitting (EF) techniques are introduced to improve the behaviour of some classical methods when some information on the solutions is known in advance. Typically, these EF methods are evaluated by computing their accuracy for some test problems and their stability behaviour is assumed to be similar to their underlying classical methods. The aim of this note is to show that in some standard explicit Runge-Kutta methods the stability of their exponentially fitted versions depends strongly on the choice of the fitting space and must be careful selected in order to assess the quality of the integrators for certain types of IVPs.

This is a joint work with J.I. Montijano, L. Rández and M. Van Daele

January 24

	Plenary room	
9:30-10:30	C.Brezinski	
	Room 1	Room 2
10:30-11:00	M. Redivo	Aidarous
11:00-11:30	A. Bultheel	Spalević
11:30-12:00	Break	
12:00-13:30	Open Problems Session)	
13:30-15:30	Lunch	

The simplified topological ε-algorithm

Michela Redivo Zaglia
University of Padova

Abstract

When a sequence of numbers is slowly converging, it can be transformed into a new sequence which, under some assumptions, could converge faster to the same limit. One of the most well–known sequence transformation is Shanks' transformation (1955) which can be recursively implemented by the ε–algorithm of Wynn (1956). This transformation and this algorithm have been extended to sequence of vectors (Wynn, 1962) or, more generally, to sequences of elements of a vector space E (Brezinski, 1975). In this talk, we present a new way of implementing the topological Shanks' transformation. It no longer requires the manipulation of elements of the algebraic dual space E^* of E, and it needs the storage of less elements of E than the topological ε–algorithm. Numerical examples will also be given.

This is a joint work with Claude Brezinski.

The Sobolev-type Infinite Hankel Matrix

Eman Salem Al-Aidarous
King Abdulaziz University

Abstract

Let $\mathbb{P}$ be the linear space of polynomials in one variable with real coefficients and let $\{\mu_i\}_{i=1}^N$ be a set of positive Borel measures supported on the real line. Let consider the following inner product in $\mathbb{P}$

$$< p, q >_S \stackrel{\text{def}}{=} \int p(x)q(x)\, d\mu_0(x) + \sum_{i=1}^{m} \lambda_i \int p^{(i)}(x)q^{(i)}(x)\, d\mu_i(x),\ \lambda_i \in \mathbb{R}^+ \quad (2)$$

and

$$< p, q >_{\mu_i} \stackrel{\text{def}}{=} \int p(x)q(x)\, d\mu_i(x),\ i = 1, \cdots, m. \quad (3)$$

Let $\overline{H}(S)$ and $\overline{H}(\mu_i)$ be the infinite moment matrices associated with the inner products (2) and (3), respectively. Notice that the first one is a structured matrix and the other ones are Hankel matrices. Then the relation between the above infinite matrices is given. Furthermore, a connection with the Sobolev moment problem is stated. Thus we extend some previous results obtained in [1] and [2] . A matrix interpretation of the sequence of orthogonal polynomials with respect to (2) and (3) in terms of the Cholesky factorization of $\overline{H}(S)$ and $\overline{H}(\mu_i)$ is presented.

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[2] F. Marcellán, F. H. Szafraniec, *A matrix algorithm towards solving the moment problem of Sobolev type.* Linear Algebra Appl. 331 (2001), no. 1-3, 155–164.

Holomorphic functions associated with indeterminate rational moment problems

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Abstract

We consider indeterminate rational moment problems on the real line with their associated orthogonal rational functions. There exists a Nevanlinna type parameterization relating to the problem, with associated Nevanlinna matrices of functions having singularities in the closure of the set of poles of the rational functions belonging to the problem. We prove that the growth at the isolated singularities of the four functions in the Nevanlinna matrix is the same.

Joint work with E. Hendriksen and O. Njåstad.

Error estimates of Gauss-Turán quadratures

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Abstract

A survey of our recent results on the error of Gauss-Turán quadrature formulae for functions which are analytic on a neighborhood of the set of integration is given. In particular, a computable upper bound of the error is presented which is valid for arbitrary weight functions. A comparison is made with the exact error and number of numerical examples, for arbitrary weight functions, are given which show the advantages of using such rules as well as the sharpness of the error bound. Asymptotic error estimates when the number of nodes in the quadrature increases are presented. A couple of numerical examples are included.

Posters

Regularized Sinc Collocation Method Applied to First kind

Neggal Bilel

Abstract

In this study one of the new techniques is used to solve numerical problems involving integral equations known as regularized sinc-collocation method. This method has been shown to be a powerful numerical tool for finding accurate solutions. So, in this talk, some properties of the regularized sinc-collocation method required for our subsequent development are given and are utilized to reduce integral equation of the first kind to some algebraic equations. Then by a theorem we show error in the approximation of the solution decays at an exponential rate. Finally, numerical examples are included to demonstrate the validity and applicability of the technique.

Asymptotics for Laguerre-Sobolev type orthogonal polynomials modified within their oscillatory regime

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Universidad Carlos III de Madrid

Abstract

In this work we consider sequences of polynomials orthogonal with respect to the discrete Sobolev inner product

$$\langle f, g\rangle_S = \int_0^\infty f(x)g(x)x^\alpha e^{-x}dx + \mathbb{F}(c)A\mathbb{G}(c)^t, \alpha > -1,$$

where f and g are polynomials with real coefficients, $A \in \mathbb{R}^{(2,2)}$ and the vectors $\mathbb{F}(c)$, $\mathbb{G}(c)$ are

$$A = \begin{pmatrix} M & 0 \\ 0 & N \end{pmatrix}, \quad \mathbb{F}(c) = (f(c), f'(c)) \text{ and } \mathbb{G}(c) = (g(c), g'(c)), \text{ respectively,}$$

with $M, N \in \mathbb{R}_+$ and the mass point c is located inside the oscillatory region for the classical Laguerre polynomials. We focus our attention on the representation of these polynomials in terms of the classical Laguerre polynomials and deduce the coefficients of their corresponding five-term recurrence relation as well as the asymptotic behavior of these coefficients when the degree of the polynomials tends to infinity. Also, the outer relative asymptotics of orthogonal polynomials with respect to this discrete Sobolev inner product is analyzed.

Two methods for interpolation of order two on the circle

Elías Berriochoa
Universidad de Vigo

Abstract

This contribution is devoted to study Hermite interpolation problems on the unit circle. The interpolation conditions prefix the values of the polynomial and its first two derivatives at the nodal points and the nodal system is constituted by complex numbers equally spaced on the unit circle. We solve the problem in the space of Laurent polynomials by giving two different expressions for the interpolation polynomials. The first one is given in terms of the natural basis of Laurent polynomials and the remarkable fact is that the coefficients can be computed in an easy and efficient way by means of the Fast Fourier Transform. The second expression is a barycentric formula, which is very suitable for computational purposes.

Hermite interpolation with extended Chebyshev nodal systems

Alicia Cachafeiro
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Abstract

The Chebyshev nodal systems play an important role in the theory of Hermite interpolation on the interval $[-1, 1]$. For the cases of nodal points corresponding to the Chebyshev polynomials of the second kind $U_n(x)$, the third kind $V_n(x)$ and the fourth kind $W_n(x)$, it is usual to consider the extended systems, that is, to add the endpoints -1 and 1 to the nodal system related to $U_n(x)$, to add -1 to the nodal system related to $V_n(x)$ and to add 1 to the nodal system related to $W_n(x)$. The interpolation methods that are usually used in connection with these extended nodal systems are quasi-Hermite interpolation and extended Hermite interpolation, and it is well known that the performance of these two great methods is quite good when it comes to continuous functions.
This work attempts to complete the theory concerning these extended Chebyshev nodal systems. For this, they have been designed new algorithms for the computation of the Hermite interpolation polynomials based upon barycentric formulas. The feature of this approach is that the derivatives of the function at the endpoints of the interval are also used. Further, some convergence results are obtained for these extended interpolants when apply to continuous functions.

Mehler–Heine Asymptotics of a Class of Generalized Hypergeometric Polynomials

Juan José Moreno Balcázar
Universidad de Almería

Abstract

We obtain a Mehler–Heine type formula for a class of generalized hypergeometric polynomials. We illustrate the results with numerical experiments and some figures.

Joint work with Cleonice F. Bracciali, Departamento de Matemática Aplicada, UNESP - Univ. Estadual Paulista, SP, Brazil.

From Shannon's sampling theory to regular and irregular U-invariant sampling

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Abstract

The classical Whittaker-Shannon-Kotel'nikov theorem states that any function with compact supported Fourier transform is completely determined by its ordinates at a series of equally spaced points. This revolutionary result has had an enormous impact due to its applications in many many branches of applied mathematics. Nowadays signals are assumed to belong to some shift-invariant subspace of $L^2(\mathbb{R})$, besides, in many common situations the available data of a signal are samples of some filtered versions of the signal itself. This leads to the problem of generalized sampling in shift-invariant spaces, i.e., to recover any function in this subspaces by means of its samples. A more general problem is to consider subspaces of a Hilbert space generated by an unitary operator U. The goal of this work is to give a survey on the history of the WSK theorem and conclude with some results in regular and irregular U-invariant sampling.

Zeros of row sequences of multi-point Padé approximants

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Abstract

The classical Jentzsch-Szegő theorem on zeros of Taylor polynomials has been extended, among other situations, to row sequences of Padé approximants [4], rational functions of best uniform approximation [2], and Padé approximants with unbounded number of poles [1]. In all cases the zero limit distribution of the approximants turns out to be the equilibrium measure of a certain set. The authors extend the Jentzsch-Szegő theorem to the case of a function f interpolated by multi-point Padé approximants along an arbitrary table of points, lying on a compact set on a neighborhood of which f is analytic. In this case the limit distribution of the zeros is not longer an equilibrium measure but it is given by a generalized balayage measure depending on the interpolation points and the region of analyticity of the function f. This result constitutes an extension to Padé approximants of a previous work [3] dealing with interpolating polynomials.

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Multiresolution analysis for bidimensional interpolatory non-uniform schemes

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Abstract

In [1] the authors propose to extend Harten's interpolatory multiresolution representation to include Hermite interpolation over an uniform partition of the unit interval $[0, 1]$. The authors develop decomposition and reconstruction algorithms and, as application, they show the compression features of the proposed method. In the present work we extend the ideas of Beam and Warming to a non-uniform two-dimensional frame. That is, we study a double generalization of the multiresolution scheme developed in [1]: on the one hand we consider the extension to the bidimensional case and, on the other one, we handle with the non uniform case by considering partitions into non-uniform grids. In this context, we consider different interpolatory schemes, we develop the corresponding algorithms and we apply them in several examples to check the higher degree of accuracy in data compression and in discontinuities detection.

This a joint work with M.A. Fortes, M. L. Rodríguez, Dept. Applied Mathematics University of Granada, Spain

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Zeros of orthogonal polynomials generated by a Geronimus perturbation of measures

Edmundo J. Huertas Cejudo
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Abstract

In the last years some attention has been paid to the so called canonical spectral transformations of measure supported on the real line. Our contribution is focused on the behaviour of zeros of MOPS associated with the Geronimus transformations of such measures. We analyze the behavior of the zeros when the intensity of the perturbation N goes from zero to infinity. Also we apply our results to the Laguerre and Jacobi classical measures.

On perturbed Szego recurrences

Kenier Castillo
Universidade Estadual Paulista

Abstract

The purpose of the present contribution is to investigate the effects of finite modifications of Verblunsky coefficients on Szego recurrences. More precisely, we study the structural relations and the corresponding C-functions of the orthogonal polynomials with respect to these modifications from the initial ones.

A measure of the overlapping of two densities: The Jensen-Fisher divergence

Alejandro Zarzo
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Abstract

The measure of Jensen–Fisher divergence between probability distributions is introduced and its theoretical grounds set up. This quantity, in contrast to the remaining Jensen divergences, grasps the fluctuations of the probability distributions because it is controlled by the (local) Fisher information, which is a gradient functional of the distribution. So it is appropriate and informative when studying the similarity of distributions, mainly for those having oscillatory character. The new Jensen–Fisher divergence shares with the Jensen–Shannon divergence the following properties: non–negativity, additivity when applied to an arbitrary number of probability densities, symmetry under exchange of these densities, vanishing under certain conditions, and definiteness even when these densities present non-common zeros. Moreover, the Jensen–Fisher divergence is shown to be expressed in terms of the relative Fisher information as the Jensen–Shannon divergence does in terms of the Kullback–Leibler or relative Shannon entropy. Finally, the usefulness of the Jensen–Fisher divergence is illustrated in some particular examples.

On the Darboux transformations for banded matrices

Dolores Barrios
Universidad Politécnica de Madrid

Abstract

In this work the concepts of Darboux factorization and Darboux transformations for arbitrary Hessenberg banded matrices are analyzed. Specifically, the existence of this kind of factorization is studied, and some sufficient conditions for the uniqueness are determined. The Darboux transformations provide a method for obtaining solutions of some integrable systems. Moreover, these transformations have applications to some relevant items of approximation theory. In particular, our work contains the key for extensions of results related to Darboux transformations to families of polynomials generated by high order recurrence relations. This is a joint work with D. Manrique.

On some properties of q-Charlier multiple orthogonal polynomials

Andys Marcos Ramirez Aberasturis
Universidad Carlos III de Madrid

Abstract

This contribution deals with multiple orthogonal polynomials of type II with respect to q-discrete measures (q-Charlier measures). In addition, we show that this family of multiple orthogonal polynomials has a raising operator as well as a Rodrigues-type formula. A high-order linear q-difference equation with polynomial coefficients having q-Charlier multiple orthogonal polynomials as eigenfunctions is obtained. Moreover, an explicit expression of the recurrence relation for these q-multiple orthogonal polynomials will be given.

This is a joint work with J. Arvesú[a] and A. Soria-Lorente[b].

Wavelet Techniques and Financial Data

Concepción González-Concepción
Universidad de La Laguna

Abstract

In this poster we illustrate the use of wavelet multiresolution decomposition and cross-wavelets analysis to reveal certain properties in financial data related to ortgages to households and gross domestic product data in Spain. We present several graphics and figures which have been obtained from the computational process addressed in a previous paper. Accordingly, we can show the usefulness of wavelet technique, which uses both time and frequency domains, as a vehicle for analysing economic and financial data. Relevant information on the different phases through which the studied variables evolve can be obtained.

This is a joint work with M.C. Gil-Fariña, C. Pestano-Gabino.

Three term relations for bivariate Koornwinder orthogonal polynomials

Teresa E. Pérez
Universidad de Granada

Abstract

In 1975, T. Koornwinder introduced a method to generate bivariate orthogonal polynomials by using orthogonal polynomials in one variable. In this work, we study the explicit expressions for the matrix coefficients in their three term relations by using the the three term recurrence relations for the involved univariate orthogonal polynomials. Moreover, some nice examples are considered.

This is a joint work with Misael E. Marriaga.

Inversion formulas and polynomial estimates for interpolation by Hankel translates of a basis function

Cristian Arteaga
Universidad de La Laguna

Abstract

For $\mu \geq -1/2$, the authors have developed elsewhere a scheme for interpolation by Hankel translates of a basis function Φ in certain spaces of continuous functions Y_n ($n \in \mathbb{N}$) depending on a weight w. The functions Φ and w are connected through the distributional identity $t^{4n}(h'_\mu \Phi)(t) = 1/w(t)$, where h'_μ denotes the generalized Hankel transform of order μ. Now we use the projection operators associated with an appropriate direct sum decomposition of the Zemanian space $\mathcal{H}_\mu$ in order to derive explicit representations of the derivatives $S_\mu^m \Phi$ and their Hankel transforms, the former ones being valid when $m \in \mathbb{Z}_+$ is restricted to a suitable interval for which $S_\mu^m \Phi$ is continuous. Here, S_μ^m denotes the m-th iterate of the Bessel differential operator S_μ^m if $m \in \mathbb{N}$, while S_μ^0 is the identity operator. These formulas, which can be regarded as inverses of generalizations of the equation $(h'_\mu \Phi)(t) = 1/t^{4n} w(t)$, allow us to get some polynomial bounds for such derivatives. Corresponding results are obtained for the members of the interpolation space Y_n.

This is a joint work with Isabel Marrero.

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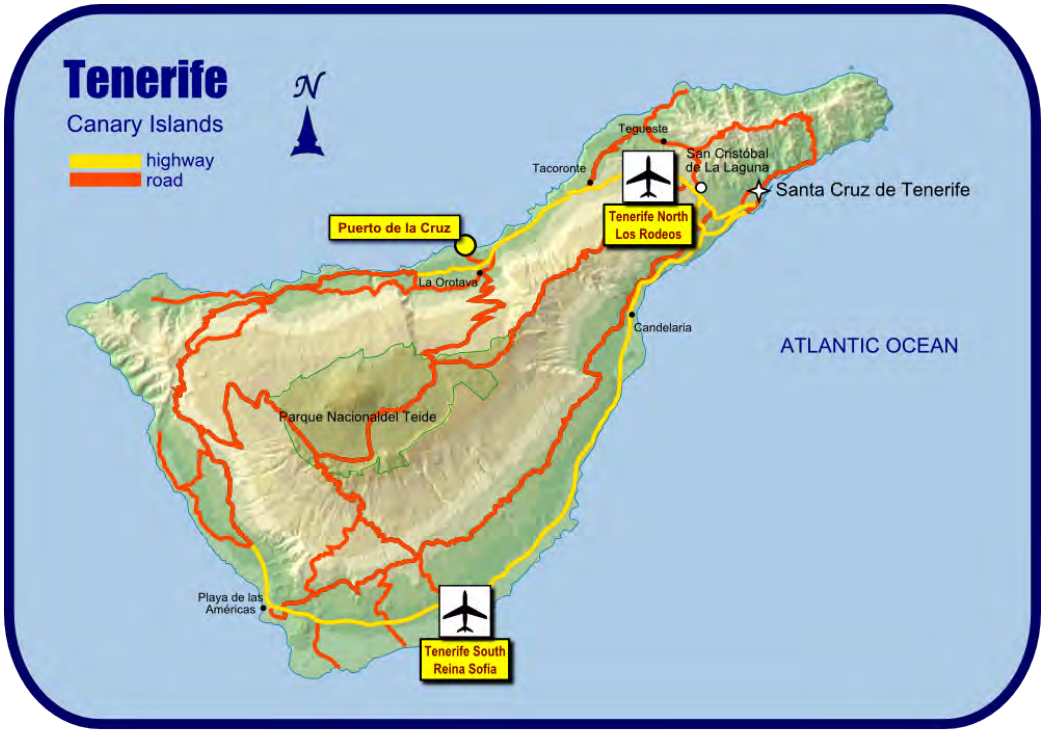
Local Map

Tenerife Island



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